Internet Appendix: A Simple Model

Consider a firm that produces two products, Product A and Product B. In this appendix, we will examine how its ROA can be affected when it adopts a technology diversity strategy or not with respect to natural disaster risks.

Production process. The firm has two production lines (Line A and Line B) dedicated to producing Product A and Product B, respectively. We assume that to produce one unit of output requires one unit of input (modelled by inventory in the firm) and one unit of capacity.

When the firm does not adopt a technology diversity strategy, then the production line of Product B can use neither the capacity nor the inventory of Line A. We therefore only consider a symmetric case for production lines for the sake of simplicity. The capacity for Line A and Line B are both K, and the firm holds I unit of inventory for Product A and Product B, respectively. The production cost per unit (including the cost of inventory and capacity) is c.

A technology diversity strategy, if adopted, is associated with the following costs and benefits. First, we assume that the initial cost to adopt a technology diversity strategy is T. Second, technology diversity enables Line A to produce Product B at an additional cost δ (and vice versa); we call switch of production "cross-production." Finally, when technology is diversified, the products may pool raw materials or work-in-process together. For example, firms can use the same raw materials or components to produce different products. In this case, we consider all $2I$ units of inventory that can be used to produce either Product A or Product B.

Product Demand. The demand for Product A (Product B) is modelled by a linear demand function, $p_A = \alpha_A - \beta_A q_A$ ($p_B = \alpha_B - \beta_B q_B$), in which p_A and p_B represent the prices, whereas q_A and q_B represent the production quantities. For simplicity's sake, we consider $\alpha_A = \alpha_B \equiv \alpha$ (i.e., the total market size is the same for both products) but $\beta_B > \beta_A$ to capture that consumers are more price sensitive to Product B than Product A. The firm needs to decide the production quantities, q_A and q_B.

Risk of Natural Disasters. Both production lines face the same risk of being hit by natural disasters. Let x be zero or one: it is zero if the natural disaster does not hit the firm, and is one otherwise. The extent that the production lines are damaged by a natural disaster (assuming that the firm is struck by the disaster) is modelled by a random variable $\pi \in [0, 1]$. The extent to which the inventory is affected (e.g., damaged by floods) is modelled by $\mu \in [0, 1]$. Therefore, the available capacity is represented by $(1-x\pi)K$, and the available inventory is represented by $(1 - x\mu)I$.

Profit Maximization. Denote the profit for a firm that does not adopt a technology diversity strategy as Π_O and the profit for a firm that does adopt a technology diversity strategy as Π_T. We find the optimal production decision for both firms by maximizing the respective profits.

Consider the firm that does not adopt a technology diversity strategy. The maximization problem becomes,

$$\begin{aligned} \max \Pi_O &= (p_A - c)q_A + (p_B - c)q_B \\ \text{st. } q_A &\leq (1 - x\pi)K, \text{ and } q_B \leq (1 - x\pi)K, \\ q_A &\leq (1 - x\mu)I, \text{ and } q_B \leq (1 - x\mu)I. \end{aligned} \tag{1}$$

The four conditions show that the produced quantity cannot be higher than the available capacity and inventory. As a side note, the optimal solution for an unconstrained problem (without the conditions on q_A and q_B) can be reduced to the solution of the traditional problem, $\max(P_A - c)q_A$ and $\max(P_B - c)q_B$, and they are $q_A^* = (\alpha - c)/2\beta_A$ and $q_B^* = (\alpha - c)/2\beta_B$, respectively.

For the firm that does adopt a technology diversity strategy, the maximization problem is more complicated. The total production quantity for Product A is q_A, which includes the quantity of Product A produced by Line A, q_{AA}, and that produced by Line B, q_{AB}, at the additional cost δ. Therefore, the maximization problem is,

$$\begin{aligned} \max \Pi_T &= (p_A - c)q_A - \delta q_{AB} + (p_B - c)q_B - \delta q_{BA} \\ \text{st. } q_A &\le (1 - x\pi)K,\ q_B \le (1 - x\pi)K, \text{ and } q_A + q_B \le 2(1 - x\mu)I, \end{aligned} \tag{2}$$

in which $q_A \equiv q_{AA} + q_{AB}$ and $q_B \equiv q_{BB} + q_{BA}$. The optimal solution for the unconstrained problem is the same as the one for (1): when the capacity and inventory are both sufficient, the firm will not utilize the cross-production capacity as it involves an extra cost of δ. In this case, $q_A^* = (\alpha - c)/2\beta_A$ and $q_B^* = (\alpha - c)/2\beta_B$, and $q_{AB} = q_{BA} = 0$. Note that we exclude the cost of adopting the technology diversity strategy, T, in the maximization, as this constant does not impact optimal quantity decisions, but rather only impacts overall profits. Therefore, the final profit is $\Pi_T - T$.

Next, we consider the ROA for both cases. ROA is calculated as the ratio of profits to assets (including capacity and inventory) in the beginning of the period. Regardless of whether a firm adopts a technology diversity strategy or not, the assets are the same for both cases; hence, to investigate how technology diversity affects a firm's operating performance under natural disaster risks, we only need to study the order of the profit difference, $\Pi_T^* - \Pi_O^*$, and the adoption cost, T

First, we show that $\Pi_T^* \ge \Pi_O^*$. It is easy to see that the optimal solution to (1) is a feasible solution to (2), as (2) is a relaxation of (1) by adding two degrees of freedom, q_{AB} and q_{BA}, and by relaxing the constraints $q_A \le (1 - x\mu)I$ and $q_B \le (1 - x\mu)I$ to only $q_A + q_B \le 2(1 - x\mu)I$. As a result, the optimal profit for the firm with a technology diversity strategy, Π_T^*, is always no lower than the one without such a strategy (i.e., Π_O^*).

Second, as a technology diversified firm incurs a cost T, we need to understand when a technology diversified strategy can benefit a firm by exploring whether $\Pi_T^* - \Pi_O^*$ is larger than T or not. To understand the impact of various channels on the profit difference, we focus on the following two scenarios: changing $x\mu$ under the condition that $x\pi \le 1 - (\alpha - c)/(2K\beta_A)$, and changing $x\pi$ under the condition that $x\mu \le 1 - (\alpha - c)/(2I\beta_A)$. The former scenario

($x\pi \leq 1-(\alpha-c)/(2K\beta_A)$) implies that the maximum capacity needed to produce optimal quantities, which is $(\alpha-c)/2\beta_A$, does not exceed the remaining capacity, $(1-x\pi)K$. Then, depending on the impact on inventory (i.e., changing $x\mu$), we can see how the remaining inventory affects the profit difference in Proposition 1. The latter scenario ($x\mu \leq 1-(\alpha-c)/(2I\beta_A)$) implies that the maximum inventory needed to produce optimal quantities, which is $(\alpha-c)/2\beta_A$, does not exceed the remaining inventory, $(1-x\mu)I$. Then, we can see how the remaining capacity affects the profit difference by changing $x\mu$ in Proposition 2. Although the entire parameter spectrum includes more combinations than just these two, these other combinations are simply more complicated versions of these two scenarios and does not provide further insights. Thus, we only discuss these two scenarios for brevity's sake.

Proposition 1 *Consider $x\pi \leq 1-(\alpha-c)/(2K\beta_A)$ and denote $\widetilde{I}=(1-x\mu)I$. The optimal profit difference $\Pi_T^*-\Pi_O^*$ can be categorized into one of the four regions:*

Region	$\Pi_T^*-\Pi_O^*$
(I): $x\mu \leq 1-\frac{\alpha-c}{2I\beta_A}$	0
(II): $1-\frac{\alpha-c}{2I\beta_A} < x\mu \leq 1-\frac{\alpha-c}{4I}\left(\frac{1}{\beta_A}+\frac{1}{\beta_B}\right)$	$\frac{(\alpha-c)^2}{4\beta_A}-(\alpha-c)\widetilde{I}+\beta_A\widetilde{I}^2$
(III): $1-\frac{\alpha-c}{4I}\left(\frac{1}{\beta_A}+\frac{1}{\beta_B}\right) < x\mu \leq 1-\frac{\alpha-c}{2I\beta_B}$	$-\frac{(\alpha-c)^2}{4\beta_B}+(\alpha-c)\widetilde{I}-\frac{(3\beta_B-\beta_A)\beta_A}{\beta_A+\beta_B}\widetilde{I}^2$
(IV): $1-\frac{\alpha-c}{2I\beta_B} < x\mu \leq 1-\frac{\alpha-c}{2I\beta_B}$	$\frac{(\beta_B-\beta_A)^2}{\beta_A+\beta_B}\widetilde{I}^2$

Figure 1 illustrates Proposition 1 by showing that if the impact on inventory due to natural disasters is low (i.e., in Region (I)), the benefit of inventory pooling will not be realized, as there is still ample inventory available. However, when the risk of natural disaster ($x\mu$) increases, the profit difference becomes an increasing convex function of $x\mu$ in Regions (II) and (IV), but becomes a concave function with the maximum occuring within the range of Region (III). Finally, when $x\mu=1$, as can be seen from the profit difference in Region (IV), $\Pi_T^*-\Pi_O^*$ becomes zero again.

Comparing the profit difference, $\Pi_T^*-\Pi_O^*$, with the cost of adopting techonology diversity, T (as shown in the dashed line), we can see that when $\Pi_T^*-\Pi_O^*$ lays above the dashed line

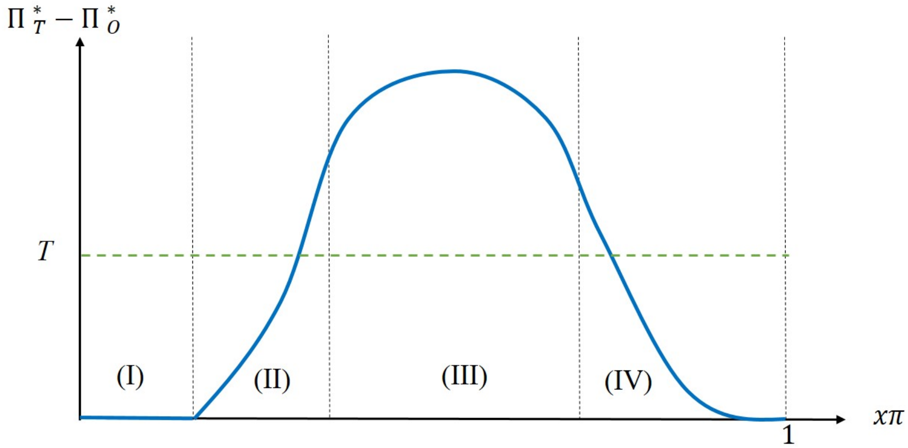

Figure 1: Change of $\Pi_T^* - \Pi_O^*$

(i.e., $\Pi_T^* - \Pi_O^* > T$), then a firm with technology diversity performs better than one without such diversity, and this can only happen when $x\mu$ is neither too small (when inventory pooling has a small impact) nor too large (when there is no sufficient inventory to be pooled in production). This proposition shows how the channel of inventory pooling can play a role under the presence of natural disaster risks. Also, when T is lower, the likelihood of $\Pi_T^* - T > \Pi_O^*$ is higher.

Proposition 2 *Consider $x\mu \leq 1 - (\alpha - c)/(2I\beta_A)$. Denote $\widetilde{K} = (1 - x\pi)K$ and*

$$\overline{x\pi} = \min\left(1 - \frac{\delta}{2K\beta_A}, 1 - \frac{\delta}{2K\left(\beta_B - \beta_A\right)}\right).$$

If $\delta < (\alpha - c)\left(\beta_B - \beta_A\right)/\beta_B$, the optimal profit difference $\Pi_T^* - \Pi_O^*$ can be categorized into *one of the five regions:*

Region	$\Pi_T^* - \Pi_O^*$
(I): $x\pi \leq 1 - \frac{\alpha-c-\delta}{2K\beta_A}$	0
(II): $1 - \frac{\alpha-c-\delta}{2K\beta_A} < x\pi \leq 1 - \frac{\alpha-c-\delta}{4K\beta_A} - \frac{\alpha-c}{4K\beta_B}$	$\frac{(\alpha-c-\delta)^2}{4\beta_A} - (\alpha-c)\widetilde{K} + \beta_A^2\widetilde{K}$
(III): $1 - \frac{\alpha-c-\delta}{4K\beta_A} - \frac{\alpha-c}{4K\beta_B} < x\pi \leq 1 - \frac{\alpha-c}{2K\beta_B}$	$-\beta_A\frac{3\beta_B-\beta_A}{\beta_A+\beta_B}\widetilde{K}^2 + \left[(\alpha-c) - \delta\frac{\beta_B-\beta_A}{\beta_A+\beta_B}\right]\widetilde{K} + \frac{\delta^2}{4(\beta_A+\beta_B)} - \frac{(\alpha-c)^2}{4\beta_B}$
(IV): $1 - \frac{\alpha-c}{2K\beta_B} < x\pi \leq \overline{x\pi}$	$\frac{(\beta_B-\beta_A)^2}{\beta_A+\beta_B}\widetilde{K}^2 - \delta\frac{\beta_B-\beta_A}{\beta_A+\beta_B}\widetilde{K} + \frac{\delta^2}{4(\beta_A+\beta_B)}$
(V): $\overline{x\pi} < x\pi \leq 1$	0

But if $\delta \geq (\alpha-c)(\beta_B-\beta_A)/\beta_B$, *then* $\Pi_T^* - \Pi_O^* = 0$, *regardless of the range of* $x\pi$.

Similar to Figure 1, Figure 2 shows the shape changes for Proposition 2. This scenario represents the case when the additional cost for cross-production (i.e., δ) is small.[1] This proposition shows how the channel of cross-production can play a role under the presence of natural disaster risks. Also, when T is lower, the likelihood of $\Pi_T^* - T > \Pi_O^*$ is higher.

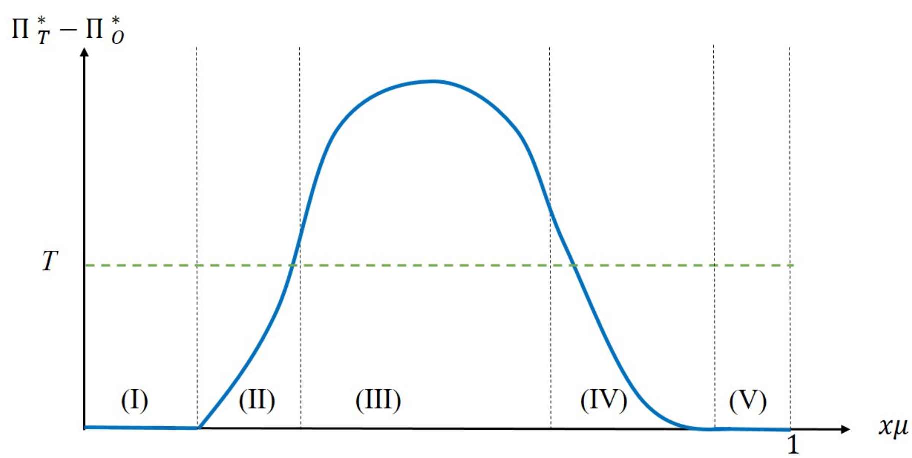

Figure 2: Change of $\Pi_T^* - \Pi_O^*$

[1]When this additional cost is too high ($\delta \geq (\alpha-c)(\beta_B-\beta_A)/\beta_B$), then cross-production will be too costly to be utilized by the firm (i.e. $\Pi_T^* = \Pi_O^*$).

Proof of Proposition 1. First, as the capacity is not constrained, the cross-production quantities will always be zero (i.e., $q_{AB} = q_{BA} = 0$). In the following, to separate the optimal quantities, we denote (q^*_{AO}, q^*_{BO}) as the production quantities for the firm without technology diversity, whereas we denote (q^*_{AT}, q^*_{BT}) as the production quantities for a firm that does have technology diversity.

In Region (I), it is easy to see that,

$$q^*_{AO} = \frac{\alpha - c}{2\beta_A} = q^*_{AT}, \text{ and } q^*_{BO} = \frac{\alpha - c}{2\beta_B} = q^*_{BT},$$

as neither capacity nor inventory are constrained, and hence $\Pi^*_T - \Pi^*_O = 0$.

In Region (II), the previous quantities remain optimal for (2), but the dedicated inventory for Product A is not sufficient for (1); hence, the firm can only process the inventory that it has on hand,

$$q^*_{AO} = (1 - x\mu)I, \text{ and } q^*_{BO} = \frac{\alpha - c}{2\beta_B},$$

leading to the profit difference in the proposition.

In Region (III), (q^*_{AO}, q^*_{BO}) remains optimal for (1), but for (2), the quantity constraint will be binding, and hence can be rewritten as,

$$\begin{aligned} \max \Pi_T &= (p_A - c)q_A + (p_A - c)q_B \\ \text{st. } q_A + q_B &= 2(1 - x\mu)I. \end{aligned}$$

By substituting the binding constraint, $q_A + q_B = 2(1 - x\mu)I$, into the profit function, we obtain,

$$\Pi_T = 2(\alpha - c)(1 - x\mu)I - 2\beta_B(1 - x\mu)I + 4\beta_B(1 - x\mu)Iq_A - (\beta_A + \beta_B)(q_A)^2.$$

Using the first-order condition, we then have the following optimal solution,

$$q^*_{AT} = \frac{2\beta_B(1-x\mu)I}{\beta_A+\beta_B}, \text{ and } q^*_{BT} = \frac{2\beta_A(1-x\mu)I}{\beta_A+\beta_B}.$$

At this point, we can see that the optimal price for the two products are the same, and the optimal profit is

$$2(1-x\mu)I\left((\alpha-c) - \frac{2\beta_A\beta_B(1-x\mu)I}{\beta_A+\beta_B}\right),$$

thus leading to the profit difference in the proposition.

Finally, in Region (IV), (q^*_{AT}, q^*_{BT}) remains to be optimal, but because both inventory piles are not sufficient to sustain the production for both A and B, we obtain

$$q^*_{AO} = (1-x\mu)I, \text{ and } q^*_{BO} = (1-x\mu)I,$$

leading to a profit of

$$\Pi^*_O = 2\left(\alpha - c\right)(1-x\mu)I - \left(\beta_A+\beta_B\right)(1-x\mu)^2 I^2,$$

and thus the profit difference in the proposition.

Proof of Proposition 2. First, we know that one of the cross-production quantities will always be zero (i.e., if $q_{AB} > 0$ then $q_{BA} = 0$, and vice versa). Moreover, because the price sensitivity of Product B is higher, we know that $q_{BA} = 0$.

Next, we analyze the firm with technology diversity. The optimization in this case can be rewritten as

$$\begin{aligned} \max \Pi_T &= (p_A - c)q_A - \delta q_{AB} + (p_B - c)q_B \\ \text{st. } q_A &\leq (1-x\pi)K \text{ and } q_B \leq (1-x\pi)K. \end{aligned}$$

If cross-production is not optimal to use (i.e., $q_{AB} = 0$), then this optimization will be reduced to (1), and hence $\Pi_T^* - \Pi_O^* = 0$. Therefore, we next find the range of parameters that allows for cross-production.

When the capacity is not sufficient (i.e., $x\pi > 1 - (\alpha - c)/(2K\beta_A)$), then it is always optimal to have $q_A = (1 - x\pi)K + q_{AB}$. For Product B, we consider two cases: (a) $(1 - x\pi)K - q_{AB} \geq (\alpha - c)/(2\beta_B)$ and (b) $(1 - x\pi)K - q_{AB} < (\alpha - c)/(2\beta_B)$. For case (a), the maximization problem becomes,

$$\max \Pi_T = (p_A - c)q_A - \delta q_{AB} + \frac{(\alpha - c)^2}{4\beta_B},$$

and $q_{BT}^* = (\alpha - c)/(2\beta_B)$. By substituting $q_A = (1 - x\pi)K + q_{AB}$, we obtain,

$$\max \Pi_T = ((\alpha - c) - \beta_A(1 - x\pi)K - \beta_A q_{AB})(1 - x\pi)K + ((\alpha - c - \delta) - \beta_A(1 - x\pi)K - \beta_A q_{AB})q_{AB} + \frac{(\alpha - c)^2}{4\beta_B}.$$

Taking the first derivative with respect to q_{AB}, we have the first-order condition as,

$$q_{AB} = \frac{\alpha - c - \delta}{2\beta_A} - (1 - x\pi)K,$$

leading to the optimal profit,

$$\Pi_T^* = \frac{(\alpha - c - \delta)^2}{4\beta_A} + \frac{(\alpha - c)^2}{4\beta_B}.$$

For this cross-production quantity to be positive, we require

$$x\pi > 1 - \frac{\alpha - c - \delta}{2K\beta_A}.$$

By substituting q_{AB} to the condition for case (a), we also require,

$$x\pi \leq 1 - \frac{\alpha - c - \delta}{4K\beta_A} - \frac{\alpha - c}{4K\beta_B}.$$

Combining all conditions together, we know cross-production capacity is only effective when,

$$1-\frac{\alpha-c-\delta}{2K\beta_A}<x\pi\leq 1-\frac{\alpha-c-\delta}{4K\beta_A}-\frac{\alpha-c}{4K\beta_B}.$$

Therefore, only when the additional cost for cross-production is low,

$$\delta<\frac{(\alpha-c)\left(\beta_B-\beta_A\right)}{\beta_B},$$

will cross-production be effective.

In this case, because the parameters satisfy the following inequality,

$$1-\frac{\alpha-c}{2K\beta_A}<1-\frac{\alpha-c-\delta}{2K\beta_A}<x\pi\leq 1-\frac{\alpha-c-\delta}{4K\beta_A}-\frac{\alpha-c}{4K\beta_B}<1-\frac{\alpha-c}{2K\beta_B},$$

we have the optimal quantities and the optimal profit for the one without technology diversity as,

$$\begin{aligned}
q_{AO}^* &= (1-x\pi)K,\ q_{BO}^*=\frac{\alpha-c}{2\beta_B}<q_{AO}^*, \text{ and}\\
\Pi_O^* &= (\alpha-c)\,(1-x\pi)K-\beta_A(1-x\pi)^2K^2+\frac{(\alpha-c)^2}{4\beta_B}.
\end{aligned}$$

Combining these results, we then have the following partial table:

Region	$\Pi_T^*-\Pi_O^*$
(I): $x\pi\leq 1-\frac{\alpha-c-\delta}{2K\beta_A}$	0
(II): $1-\frac{\alpha-c-\delta}{2K\beta_A}<x\pi\leq 1-\frac{\alpha-c-\delta}{4K\beta_A}-\frac{\alpha-c}{4K\beta_B}$	$\frac{(\alpha-c-\delta)^2}{4\beta_A}-(\alpha-c)\,(1-x\pi)K+\beta_A(1-x\pi)^2K^2$

Next, for case (b), we obtain $(1-x\pi)K-q_{BA}<(\alpha-c)/(2\beta_B)$, which is equivalent to,

$$x\pi>1-\frac{\alpha-c-\delta}{4K\beta_A}-\frac{\alpha-c}{4K\beta_B},$$

the remaining capacity of Line B is not sufficient to produce the optimal quantity for Product B. Therefore, we can rewrite the optimization to $q_{AA} = (1 - x\pi)K$, $q_{BB} = (1 - x\pi)K - q_{AB}$, and hence the optimization becomes,

$$\begin{aligned}\max \Pi_T &= (\alpha - c - \beta_A((1 - x\pi)K + q_{AB}))((1 - x\pi)K + q_{AB}) - \delta q_{AB} \\ &\quad +(\alpha - c - \beta_B((1 - x\pi)K - q_{AB}))((1 - x\pi)K - q_{AB}).\end{aligned}$$

Also, the first-order condition becomes,

$$q_{AB} = \frac{\beta_B - \beta_A}{\beta_A + \beta_B}(1 - x\pi)K - \frac{\delta}{2\left(\beta_A + \beta_B\right)}.$$

This cross-production quantity must be positive,

$$x\pi < 1 - \frac{\delta}{2K\left(\beta_B - \beta_A\right)},$$

and also results in a positive q_{BB},

$$x\pi < 1 - \frac{\delta}{2K\beta_A}.$$

In the feasible region, the optimal profit becomes:

$$\Pi_T^* = -\frac{4\beta_A\beta_B}{\beta_A + \beta_B}(1 - x\pi)^2K^2 + \left[2\left(\alpha - c\right) - \delta\frac{\beta_B - \beta_A}{\beta_A + \beta_B}\right](1 - x\pi)K + \frac{\delta^2}{4\left(\beta_A + \beta_B\right)}.$$

Therefore, we can categorize the remaining regions as the second part of the table with Regions (III) to (V).

Finally, if the extra cost of cross-production is too high,

$$\delta \geq \frac{\left(\alpha - c\right)\left(\beta_B - \beta_A\right)}{\beta_B},$$

then Region (II) will not adopt any cross-production, and hence $\Pi_T^* - \Pi_O^* = 0$. Moreover, the intersection of the condition for case (b) and the condition for cross-production,

$$1 - \frac{\alpha - c - \delta}{4K\beta_A} - \frac{\alpha - c}{4K\beta_B} < x\pi < 1 - \frac{\delta}{2K\left(\beta_B - \beta_A\right)},$$

is an empty set in this case, meaning that cross-production is not allowed and, hence, $\Pi_T^* - \Pi_O^* = 0$.