Aggregate Technology Shocks, Market Returns, and Market Premiums

Po-Hsuan Hsu

Department of Finance and Economics
Graduate School of Business
Columbia University

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‡ Correspondence: Tel: (212) 853-9178, and email: ph2048@columbia.edu.

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Abstract

This paper highlights the role of technology in asset pricing by demonstrating market return predictability based on aggregate technology shocks from both theoretical and empirical perspectives. I solve simple general equilibrium models, in which technology shocks drive conditional mean and volatility of future economic growth. The expected market returns and premiums therefore vary across time. This implication is strongly supported by empirical evidence from both U.S. and U.K. data. I use the growth of total patents and research and development (R&D) expenditures as proxies for technological growth. I then find that the technology shocks, i.e. unexpected growth of patents and R&D expenditures, have strong and distinctive explanatory power for market returns and premiums in both short- and long-term predictive regressions. These findings surpass survive robustness checks.

JEL classification: E32; E44; G12; O30

Keywords: Predictive regression; production-based asset pricing; real business cycle; return predictability; technology shocks

1 Introduction

The evidence of the economic impact of science and technology is all around us.
Zvi Griliches (1987)

Since the seminal paper of Solow (1957), the economic literature has long recognized that technologies, as observable activities that permanently improve productivity, are driving forces behind economic growth. Technology shocks have been identified and confirmed as an important source of macroeconomic fluctuations since Kydland and Prescott (1982). However, the linkage between technologies and stock returns has rarely been explored in the finance literature. Specifically, the effects of technology shocks on market returns and premiums are mixed in previous theoretical work (e.g. Lettau, 2003) and have not been empirically verified. In this paper, I demonstrate that technology shocks can drive up expected market returns and premiums in a general equilibrium setting. More importantly, this causality is substantiated by my empirical study based on patent and research and development (R&D) data. This paper therefore suggests that technology shocks may be an important component in asset pricing models.

In the first part of this paper, I construct a real business cycle model composed of one abstract good, one representative agent, and one representative firm. Despite its simplicity, this economy describes the dynamics of the financial market, labor market, representative firm's productions, and representative agent's work and consumption choices. Using a separable logarithmic utility function, I derive an exact closed-form solution to this general equilibrium model and characterizes the asset returns in terms of technology shocks and non-technology shocks. I find that time-variant technology shocks influence the productivity and alter the conditional mean and volatility of economic growth, leading to commensurate changes in expected market returns and premiums.

The economic intuition behind the model-implied market return predictability is relatively straightforward. When the budget-constrained agent observes a positive technology shock, he realizes that the shock permanently raises productivity, and his permanent income consequently increases. According to permanent income hypothesis, the agent prefers to consume more today and demands higher expected asset returns in exchange for today's consumption.[1] This predictability can be also explained through the production-side channel. If a positive technology shock occurs, the budget-constrained firm's productivity and expected investment returns rise. Since the expected market returns should equal expected investment returns in equilibrium (e.g. Cochrane, 1991), the expected market returns are higher.

The economic intuition behind the model-implied premium predictability can be explained based on economic uncertainty. A positive technology shock makes economic growth more

[1] I recognize that, in the literature, the effect of productivity-relevant shocks on marginal rate of substitution could be positive or negative, depending on model settings (e.g. Balvers and Huang, 2006).

volatile,[2] thus causing the agent to demand higher market premium in holding market portfolio (e.g. Bansal and Yaron, 2004). As a result, technology shocks should lead market premiums. The stated return predictability and premium predictability do not, however, contradict market efficiency. Rather, they simply reflect the time series variation of the stochastic discount factor and economic uncertainty caused by technology shocks.

To substantiate the proposed model and its predictions, I need measurable variables to describe the aggregate technological growth in the empirical study. I employ two proxies, the growth rates of successful patent applications and research and development (R&D) expenditures, both of which have been widely used in the economic literature.[3] Unlike R&D data, patent data has rarely been considered in the finance literature.[4] However, the total patent growth could be a purer and better proxy for aggregate technological growth because: (1) patents are the real output of R&D activities; (2) patents are ready to be utilized and have business value; and (3) the territorial principle applies to patent laws.[5] By considering both proxies, I can more precisely gauge technological growth and shocks as well as their effects on asset prices.

I consider data from both the United States and the United Kingdom in my empirical study. In the U.S. data, I find that patent shocks and R&D shocks significantly predict the raw and excess returns of the Center for Research in Security Prices (CRSP) value-weighted index and the Standard and Poor's 500 (S&P500) index for both short- and long-term horizons. Moreover, these two shocks compare favorably against other predictors including the consumption to wealth ratio, labor income to consumption ratio, relative risk-free rate, dividend to price ratio, payout ratio, term spread, and default spread. In other words, technology shocks help us to explain a distinct part of market return variation that has not been explained before. This predictability is of statistical and economic significance and survives several robustness checks. Similar results are found in the U.K. data: U.K. patent shocks predict the raw and excess returns of the Financial Times Stock Exchange 100 (FTSE100) index. My investigation therefore provides strong empirical evidence for the proposed theoretical linkage between technology shocks and expected market returns and premiums.

Finally, I consider a more general model that assumes an inseparable power utility function and allows for capital accumulation. Although there exists no closed-form solution for this model, economic dynamics can be numerically solved by a recursive value function iteration technique (Christiano, 1990a and 1990b). Consistent with earlier findings, I find that the technology shocks

[2]For example, if we unintentionally invented many technologies today, we can expect more possible outcomes of production tomorrow. So, the range of possible economic growth becomes wider.

[3]Patents and R&D expenditures have been commonly used as proxies for technologies in the literature since Griliches (1984) and Pakes (1985). Other proxies proposed in the literature include the number of scientific journal articles (Price, 1963), the number of book published (Alexopoulos, 2006), etc.

[4]Pakes (1985) and Rossi (2005) could be the only two to my limited knowledge.

[5]Generally speaking, a patent needs the approval of a national patent office to be protected in that nation's territory. Conversely, R&D can be implemented in one nation and utilized in another.

are positively correlated with future market returns and premiums in a calibrated economy. Therefore, the return and premium predictability based on technology shocks also exists in a more general economy.

The rest of the paper is organized as follows. Section 2 summarizes relevant studies and explains my contribution to the literature. In Section 3, I construct a simple economy and derive a closed-form solution based on a separable log utility function. Section 4 describes my data and discusses empirical findings and the associated robustness checks. I then consider a more general model based on an inseparable power utility function and solve it numerically in Section 5. Section 6 concludes this paper.

2 Relevant Literature

This study relates to several sets of the literature. The first is the technology and asset returns literature. There are limited empirical studies exploring the relationship between asset returns and technological activities on the firm/industry level. The earliest study in this set is Pakes (1985), who investigates the dynamics among patents, R&D costs, and stock returns on a micro data set that contains 120 firms over an 8-year period, and finds that stock returns are correlated with concurrent and lagged patents and R&D expenses. Other studies in this direction focus mainly on connecting the cross-sectional variations of stock returns with firms' research activities: Chan, Lakonishok, and Sougiannis (2001); Arora, Ceccagnoli, and Cohen (2005) discuss the R&D and patent premiums; and Apedjinou and Vassalou (2004) consider the firm's innovations and its stock returns. On the other hand, even fewer studies have been devoted to theoretical modeling of the linkage between technology and asset returns: Lettau (2003) applies Campbell's (1994) log-linear approximation to connect technology shocks to market returns and premiums. Pastor and Veronesi (2005) construct a model explaining the technological revolution process, in which a new technology is an idiosyncratic risk in the beginning and may become a systematic risk once being widely adopted. Panageas and Yu's (2006) model explains why technology shocks lead to business cycles and counter-cyclical market premiums. In this study, I try to construct a distinct and testable model, which can be empirically justified by measurable technology proxies, i.e. both patents and R&D expenses.

Since technology is an important component in the production function, the production-based asset pricing approach helps to build a theoretical relationship between technology and asset prices. Recently, some researchers in this field have started to notice the role of technology in production-based asset pricing research. For example, Belo (2005) models "technology" heterogeneity using heterogeneous production functions to explain the cross-sections of stock returns. Moreover, only a few studies in production-based asset pricing try to explain the time series variation of stock returns (Balvers, Cosimano, and McDonald, 1990; Cochrane, 1991; Rodriguez, Restoy, and Pena, 2002; Balvers and Huang, 2006), and virtually all of them use industrial production as the explanatory variable. By constructing a general equilibrium model in which the expected market returns are characterized by production variables, I can explain why the time series variation of market returns can be attributed to technology shocks.

Another set of relevant literature is market return predictability. Researchers have proposed several macroeconomic variables and financial ratios to predict market returns, and their reasons are that these variables contain valuable information pertinent to time dependent fluctuations in expected market returns due to time-varying economic conditions and investors' preferences. Financial ratios including the dividend to price ratio (Shiller, 1984; Campbell and Shiller, 1988; Fama and French, 1988), the term spread and default spread (Fama and French, 1989), the book-to-market ratio (Kothari and Shanken, 1997), the payout ratio (Lamont, 1998), and the ratio of share prices to GDP (Rangvid, 2006) have been employed to predict stock returns. On the other hand, macroeconomic variables including the relative risk-free rate (Campbell, 1990 and 1991), industrial production (Balvers, Cosimano and McDonald, 1990; Chen, 1991), aggregate consumption to wealth ratio ("*cay*" of Lettau and Ludvigson, 2001), and labor income to consumption ratio (Santos and Veronesi, 2006) have also been constructed as predictors. Since technology shock is a critical component of productivity, I was motivated to inspect its explanatory power for expected stock returns in terms of time series variations.

Finally, this paper also relates to the real business cycle research. The role of technology shocks in macroeconomic dynamics was first addressed in Kydland and Prescott's (1982) pioneering paper. Rouwenhorst (1995) connects business cycle research methods to asset pricing and constructs a general equilibrium model that illustrates the economic dynamics including the risk-free rate and stock market return. His study demonstrates the promising possibility in applying dynamic equilibrium models to develop new asset pricing implications. In addition, Rouwenhorst shows that the return predictability can be demonstrated in the simulated dynamics of a calibrated economy. His approach allows me to inspect the predictability of technology shocks in models without closed-form solutions. In this study, I demonstrate that technology drives the output growth and stock return in a calibrated economy, which further enhances our knowledge in this field.

3 The Economy and A General Equilibrium Solution

In this section, I construct an economy to describe the interactive dynamics among financial markets, production activities and consumption choices. My model setting is based on, but differs substantially from Balvers, Cosimano, and McDonald (1990), and Balvers and Huang (2006). This economy contains one representative agent, one representative firm, and one consumption good. While this economy is a simple one, it delivers an intuitive and tractable general equilibrium solution. More specifically, I can characterize the stochastic discount factor and the general equilibrium asset returns in terms of production variables. Such a characterization allows me to demonstrate why and how technology shocks affect real production and asset returns.

I first describe the basic setting and timeline of my model. I then derive the stochastic discount factor, the investment return, the market return, and the risk-free rate. Finally, I discuss the empirical implications of the proposed model.

3.1 Basic settings

The infinitely lived representative agent maximizes her/his period t time-additive expected utility as follows

$$\underset{\{n_t,\, s_{t+1},\, b_{t+1}\}}{Max} \left\{ u(c_t, \bar{n} - n_t) + \sum_{\tau=1}^{\infty} \beta^{\tau} E_t \left[u(c_{t+\tau}, \bar{n} - n_{t+\tau}) \right] \right\} \tag{1}$$

$$s.t. \quad c_t + s_{t+1}\, p_t + b_{t+1} = s_t(p_t + d_t) + b_t(1 + r_t^f) + n_t w_t, \tag{2}$$

where β is a subjective discount rate, and $u(c_t, \bar{n} - n_t)$ characterizes the agent's periodic utility function. The latter depends on the agent's consumption c_t and leisure $\bar{n} - n_t$, where $\bar{n}$ denotes total available time units and n_t denotes the labor input. s_t denotes the fractional share of the stock of the representative firm held by the agent, and d_t denotes the dividend per share distributed by the firm. p_t denotes the competitively determined stock price in time t, while b_t is the loan or debt provided by the agent, which is presumed to be risk-free. r_t^f is the risk-free rate for the period $t-1$ to t. Lastly, n_t and w_t denote the labor input and the competitive real wage, respectively.

The representative firm is operated to maximize its stock price, the total value of all discounted future dividends. The firm's maximization objective in time t is

$$\underset{\{n_t,\, k_{t+1},\, b_{t+1}\}}{Max} \left\{ d_t + \sum_{\tau=1}^{\infty} E_t \left[\left(\prod_{i=t+1}^{t+\tau} m_i \right) d_{t+\tau} \right] \right\} \tag{3}$$

$$s.t. \;\; d_t = F(n_t, k_t, A_t, \varepsilon_t) - k_{t+1} + b_{t+1} - b_t(1 + r_t^f) - n_t w_t \geq 0 \tag{4}$$

$$F(n_t, k_t, A_t, \varepsilon_t) = \alpha_0 {n_t}^{\alpha_1} k_t^{\alpha_2} {A_t}^{\alpha_3} \varepsilon_t \tag{5}$$

$$A_t = A_{t-1}\gamma_t, \quad \gamma_t = \mu\, exp\, (\xi_t), \tag{6}$$

where m_{t+1} denotes the stochastic discount factor of the investor (i.e. the agent) from time t to time $t+1$:

$$m_{t+1} = \beta \frac{\partial u(c_{t+1}, \bar{n} - n_{t+1})/\partial c_{t+1}}{\partial u(c_t, \bar{n} - n_t)/\partial c_t}. \tag{7}$$

Equation (4) essentially defines the dividend in the context of the firm's period-to-period budget constraint: d_t denotes the dividend distributed to the agent in time t, which must be larger than or equal to zero. The firm uses its production output and new debt issuances to pay dividends and wages, implement new investment, and pay off old debt with interest. The firm's production output, $F(n_t, k_t, A_t, \varepsilon_t)$, derives from a Cobb-Douglas production function that contains labor input n_t, capital input k_t, a technology component A_t, and a temporary non-technological shock ε_t in level.[6] For simplicity in notation, I use $F_t(\cdot)$ to replace $F(n_t, k_t, A_t, \varepsilon_t)$ hereafter. k_{t+1} is the output in period t reserved for investment in period $t+1$, which fully depreciates after being utilized in time $t+1$.[7] A_t denotes the period t accumulated technology level that is the compound of technological growth, γ_t, since time 0. I assume γ_t is determined by a stationary growth μ and an unexpected permanent technology shock in growth, ξ_t, which is serially uncorrelated and satisfies $E_{t-1}[exp\ (\xi_t)] = 1$.[8] I also assume that ξ_t is distributed with mean ν_ξ and variance σ_ξ^2, and $exp\ (\xi_t) \geq 1/\mu$. This is a reasonable setting and guarantees the positivity of technology level process $\{A_t\}$. The last term in the production function, ε_t, represents the unexpected non-technology shock that is i.i.d. and satisfies following conditions: $E_{t-1}[\varepsilon_t] = 1$; $\varepsilon_t \gg 0$ (non-negative output); $E_{t-1}[ln(\varepsilon_t)] = \nu_\varepsilon$. It is intended to capture all other uncertainties (e.g. oil shocks, fiscal shocks, and weather disasters technology).[9] ε_t is independent of the technology shock ξ_t and other contemporaneous variables. It can be observed that, in this model, the Solow residuals are separated into a technology component (A_t) and a non-technology component (ε_t). Such a separation is important in explaining some macroeconomic phenomena according to recent literature (e.g. Gali, 1999; Gali and Rabanal, 2004).

Some details of the technology are worth mentioning: First, like most models in the literature, I assume the "neutrality" of technology (Solow, 1957; Griliches, 1988, p.287): Technology does not change the structure of production function. Second, the obsolescence rate of technology (i.e. the depreciation of technology) is a constant and is absorbed by the μ in this study.[10] Third,

[6] Inclusion of technology in a Cobb-Douglas production function is common in the literature (e.g. Griliches, 1988, p.247). Gomulka (1990, p.52) proposes a production function including a technology component with a power parameter.

[7] I set this full depreciation assumption to simplify the model and derive a closed-form solution. However, relaxing this assumption will not alter the model implications in general.

[8] The assumption of serial uncorrelated technology shocks can be relaxed without changing the model implications.

[9] Denison (1967) could be the earliest paper to treat the productivity shock and the technical (knowledge) progress as separate components. Many recent studies in real business cycle research employ shocks other than technology shocks to explain economic fluctuations (e.g. Rebelo, 2005).

[10] Pakes and Schankerman (1984a) set a constant obsolescence rate of technology, while Abel (1984) set a stochastic one.

for analytical simplicity, the technology process is exogenous and unaffected by labor and capital input in this model.[11]

Here I recapitulate the effective timing of production variables: capital/investment needs only one period to build up; new technology invented/discovered in this period can be used in the next period; and labor input can be instantaneously adjusted. There is only one good in this economy, and all variables considered in this model are in real term (i.e. no inflation). Moreover, the main sources of risk in this model are the technology shocks, ξ, and the non-technology shocks, ε.

Here I summarize the timeline of my model:

1. In the beginning of a period, time t, two shocks occur: an unexpected permanent technology shock, ξ_t, and a temporary non-technology shock, ε_t. Both shocks are observed by the agent and the firm.

2. At the end of period t, the equilibrium wage and labor, are decided by the interaction of agent's labor supply and firm's labor demand. The firm then executes its production plan. The firm's output, $F(t)$, is used to pay the total wage bill and the old debt at the risk-free interest rate. Finally, the firm issues new debt, if necessary, implements new investment k_{t+1}, and distributes the dividend d_t. At the same time, the agent receives the dividend d_t and labor income $n_t w_t$ decides how much to consume today c_t and how much to invest in stock s_{t+1} given the current market stock price p_t, and how much debt b_{t+1} to lend to the firm at the next period given the equilibrium risk-free rate. The agent's and the firm's decisions are known by each other, and both parties share the same expectations on technological and non-technological uncertainties.

3.2 An exact closed-form solution

To initiate my analysis, I need to posit the agent's utility function. I consider the case in which the agent's utility function is logarithmic and additively separable in leisure and consumption:

$$u(c_t, \bar{n} - n_t) = \rho_1 \, ln(c_t) + \rho_2 \, ln(\bar{n} - n_t); \tag{8}$$

in this case the stochastic discount factor from time t to time $t+1$ is

$$m_{t+1} = \beta \, c_t / c_{t+1}. \tag{9}$$

The utility function is taken from Long and Plosser (1983) and Hansen (1985). Despite its simplicity, this utility function allows me to derive an exact closed-form solution and explicit

[11]This model can accommodate an endogenous technological growth by setting $\mu_t = \kappa \, k_t$. Such an endogenous setting will not alter the main model implication derived in following sections. In the literature, technological growth has been treated as an exogenous variable (e.g. Panageas and Yu, 2006).

model implications. Nevertheless, I also consider a more general model with inseparable power utility and capital accumulation in Section 5.

There exist explicit solutions for the optimal c_t and k_{t+1} policy functions, in which both are propositional to total output:

$$c_t = q\,F_t(\cdot)$$
$$k_{t+1} = (1-q)F_t(\cdot),$$

where $0 < q < 1$. These two conditions are commonly observed in the literature (e.g. Hercowitz and Sampson, 1991; Cochrane, 1996), and can be derived from the analogous social planning formulation of this model (Appendix A).[12]

The equilibrium wage w_t and labor n_t are jointly determined by the firm and the agent acting competitively. The firm's choice of labor input can be obtained by first order condition (FOC) of Equation (3) with respect to n_t:

$$\frac{\alpha_1 F_t(\cdot)}{n_t} = w_t, \tag{10}$$

which simply states that the marginal product of labor equals the wage. The agent's choice of labor given the wage rate can be solved by differentiating Equation (1) with respect to n_t, which implies

$$\begin{aligned} 0 &= u_c(c_t, \bar{n} - n_t)\frac{\partial c_t}{\partial n_t} + u_{\bar{n}-n_t}(c_t, \bar{n} - n_t) = \frac{1}{c_t}\left(s_t\frac{\partial d_t}{\partial n_t} + w_t\right) - \frac{\rho_2}{\bar{n} - n_t} \\ &= \frac{1}{q\,F_t(\cdot)}\left(s_t\frac{\alpha_1 F_t(\cdot)}{n_t} + (1 - s_t)w_t\right) - \frac{\rho_2}{\bar{n} - n_t} = \frac{\alpha_1}{q\,n_t} - \frac{\rho_2}{\bar{n} - n_t}. \end{aligned} \tag{11}$$

This first order condition (FOC) states that, at the optimum, the marginal utility of one unit of leisure should equal the marginal gain of giving up one unit of leisure, which includes wages from working and dividends from share-holding of the firm. I can therefore solve the labor input and wage as:

$$n_t = \alpha_1\bar{n}/(\rho_2 q + \alpha_1) \quad \text{and} \quad w_t = \alpha_0\alpha_1[\alpha_1\bar{n}/(\rho_2 q + \alpha_1)]^{\alpha_1 - 1} k_t^{\alpha_2} A_t^{\alpha_3} \varepsilon_t. \tag{12}$$

It is observed that the labor is a constant, which is also common in real business cycle literature (e.g. Benassy, 1995) and matches the social planner's problem described in Appendix A. It is also noted that the wage is determined by the investment, technology, and production uncertainty.

The firm's investment choice is solved by differentiating Equation (3) with respect to k_{t+1}, which is

$$E_t\left[m_{t+1}\frac{\alpha_2 F_{t+1}(\cdot)}{k_{t+1}}\right] = 1 \quad \forall\, t, \tag{13}$$

[12] This unique Pareto optimal allocation which is derived in a social planner's model must coincide with the corresponding competitive equilibrium, which can be regarded as its decentralized counterpart (Harris, 1987; Danthine and Donaldson, 2001).

where $\alpha_2 F_{t+1}(\cdot)/k_{t+1}$ denotes the investment returns and is labelled R^i_{t+1}. By imposing $c_t = q\,F_t(\cdot)$ and $k_{t+1} = (1-q)F_t(\cdot)$ into Equation (13), it can be found that $1-q = \beta\alpha_2$ and thus

$$k_{t+1} = \beta\alpha_2 F_t(\cdot); \quad c_t = (1-\beta\alpha_2)F_t(\cdot). \tag{14}$$

Here I assume the risk-free assets are in zero net supply, i.e. $\{b_t\}_{t=1}^{\infty} \equiv 0$.[13] Moreover, the dividend can be derived as

$$d_t = F_t(\cdot) - \beta\alpha_2 F_t(\cdot) - n_t w_t = (1-\beta\alpha_2-\alpha_1)F_t(\cdot), \tag{15}$$

where $n_t w_t = \alpha_1 F_t$ has been shown in Equation (10).

The final piece of this general equilibrium model is the equilibrium stock price, which can be obtained by the FOC of agent's expected utility with respect to s_{t+1}:

$$0 = \frac{\partial\left\{u(c_t, \bar{n}-n_t) + \sum_{\tau=1}^{\infty}\beta^{\tau}E_t\left[u(c_{t+\tau}, \bar{n}-n_{t+\tau})\right]\right\}}{\partial s_{t+1}},$$

$$\text{where} \quad c_t + s_{t+1}\,p_t + b_{t+1} = s_t(p_t+d_t) + b_t(1+r^f_t) + n_t w_t. \tag{16}$$

Note that the labor n_t is a constant now. Solving the above equation forward will lead to a common pricing formula:

$$p_t = \sum_{\tau=1}^{\infty} E_t\left[\left(\prod_{i=t+1}^{t+\tau} m_i\right) d_{t+\tau}\right]. \tag{17}$$

Taking the derived c_t, c_{t+1}, and d_{t+1} into Equation (17), the stock price is solved as:

$$p_t = E_t\sum_{h=1}^{\infty}\beta^h\frac{c_t}{c_{t+h}}\,d_{t+h} = \frac{\beta}{1-\beta}\,d_t. \tag{18}$$

Without loss of generality, I normalize the number of shares to one in each period (i.e. $\{s_t\}_{t=1}^{\infty} \equiv 1$). Substituting the stock price p_t and other variables derived in this section back Equation (2), the agent's budget constraint is satisfied, and so the market is cleared.

The stock returns, R^s_{t+1}, can be represented as

$$R^s_{t+1} = \frac{p_{t+1}+d_{t+1}}{p_t} = \frac{1}{\beta}\frac{d_{t+1}}{d_t} = \frac{1}{\beta}\frac{F_{t+1}(\cdot)}{F_t(\cdot)}. \tag{19}$$

The first observation is that the stock returns are determined by the dividend growth. So, any reason that causes higher dividend will lift up the stock returns. It is also clear that the investment returns exactly equal the stock returns period by period ($R^s_{t+1} = R^i_{t+1} = \alpha_2 F_{t+1}(\cdot)/k_{t+1}$), and the Euler's equation $E_t[R^s_{t+1}m_{t+1}] = 1$ holds for all periods (because $m_{t+1} = \beta c_t/c_{t+1} = \beta F_t(\cdot)/F_{t+1}(\cdot)$). The equality between market return and investment return has been shown

[13] In fact, since the firm's investment k_{t+1} is always less than its output $F_t(\cdot)$ in this model, the representative firm does not need to borrow anything from the agent.

in production-based asset pricing literature (e.g. Cochrane, 1991; Restoy and Rockinger, 1994; Zhang, 2005b).

(EXAMPLE 1) Here I use a simple case to exemplify the effect of one positive technology shock on economic dynamics. I first make the following assumptions: (1) the process of technology is set as

$$\{A_t\}_{t=1,...,T} = \underbrace{\{1, 1, 1, ..., 1}_{t=1,...,t^*-1}, \underbrace{1.5, 1.5, ..., 1.5\}}_{t=t^*,...,T},$$

which implies one half unit of shock occurring in time t^*; (2) the process of non-technology is of all ones: $\{\varepsilon_t\}_{t=1,...,T} \equiv 1$; (3) all variables decided before time t^* are in steady state, and are set as constants: F, $k = (1-q)F$, $c = q\,F$, $R^s = 1/\beta$, and the economic growth is 1; (4) n is known to be constant in all time periods. Then, I start my economic dynamics with $F_{t^*}(\cdot)$:

$$F_{t^*}(\cdot) = \alpha_0 n^{\alpha_1} k^{\alpha_2} A_{t^*}^{\alpha_3} \varepsilon_{t^*} = \alpha_0 n^{\alpha_1} k^{\alpha_2} (1.5)^{\alpha_3} = (1.5)^{\alpha_3} F$$

$$k_{t^*+1} = (1-q)F_{t^*}(\cdot) = (1-q)(1.5)^{\alpha_3} F$$

$$c_{t^*} = q\,F_{t^*}(\cdot) = q(1.5)^{\alpha_3} F$$

$$d_{t^*} = (1-\beta\alpha_2-\alpha_1)F_{t^*}(\cdot) = (1-\beta\alpha_2-\alpha_1)(1.5)^{\alpha_3} F$$

$$R_{t^*}^s = \frac{1}{\beta}\frac{F_{t^*}(\cdot)}{F} = \frac{1}{\beta}(1.5)^{\alpha_3}.$$

In the next period (t^*+1), the economic growth will be

$$F_{t^*+1}(\cdot)/F_{t^*}(\cdot) = \alpha_0 n^{\alpha_1} k_{t^*+1}^{\alpha_2} A_{t^*+1}^{\alpha_3} \varepsilon_{t^*+1}/F_{t^*}(\cdot) = \left[\frac{k_{t^*+1}}{k_{t^*}}\right]^{\alpha_2} = (1.5)^{\alpha_2\alpha_3},$$

because $k_{t^*} = k$. Note that economic growth, consumption growth, and dividend growth are all the same in this model. It is noteworthy that the capital k plays a key role in the intertemporal causality between an uncorrelated technology shock and persistent economic growth. Moreover, the stock return will be

$$R_{t^*+1}^s = \frac{1}{\beta}\frac{F_{t^*+1}(\cdot)}{F_{t^*}(\cdot)} = \frac{1}{\beta}(1.5)^{\alpha_2\alpha_3} > R^s = \frac{1}{\beta}.$$

Similarly, an uncorrelated technology shock leads persistent stock returns through its effect on capital k. So, this example clearly illustrates that a positive technology shock will drive higher economic growth and stock return in the next period.

It is also interesting to inspect the following three predictors implied by this model: the dividend to price ratio $(d-p)$, the payout ratio $(d-e)$, and the labor income to consumption ratio (SW).

$$d-p: \ \frac{d_t}{p_t} = \frac{1-\beta}{\beta},$$
$$d-e: \ \frac{d_t}{F_t(\cdot)-n_t w_t} = \frac{(1-\beta\alpha_2-\alpha_1)F_t(\cdot)}{(1-\alpha_1)F_t(\cdot)} = \frac{1-\beta\alpha_2-\alpha_1}{1-\alpha_1},$$
$$\text{SW}: \ \frac{n_t w_t}{c_t} = \frac{\alpha_1 F_t(\cdot)}{(1-\beta\alpha_2)F_t(\cdot)} = \frac{\alpha_1}{1-\beta\alpha_2}.$$

As a result, in this simplistic model, $d-p$, $d-e$, and labor income to consumption are all constants.

3.3 Model implications

The stock returns can be characterized as follows:

$$\begin{aligned} R^s_{t+1} &= R^i_{t+1} = \alpha_2 F_{t+1}(\cdot)/k_{t+1} = \frac{1}{\beta}\, g_{t+1} \\ &= \alpha_0\alpha_2 n_{t+1}^{\alpha_1} k_{t+1}^{\alpha_2-1} A_{t+1}^{\alpha_3}\varepsilon_{t+1} = \alpha_0\alpha_2[\alpha_1\bar{n}/(\rho_2 q+\alpha_1)]^{\alpha_1}[\beta\alpha_2 F_t(\cdot)]^{\alpha_2-1}A_{t+1}^{\alpha_3}\varepsilon_{t+1} \\ &= \alpha_0\alpha_2[\alpha_1\bar{n}/(\rho_2 q+\alpha_1)]^{\alpha_1\alpha_2}(\beta\alpha_0\alpha_2)^{\alpha_2-1}k_t^{\alpha_2(\alpha_2-1)}A_t^{\alpha_3(\alpha_2-1)}\varepsilon_t^{\alpha_2-1}A_{t+1}^{\alpha_3}\varepsilon_{t+1} \\ &= \alpha_0\alpha_2[\alpha_1\bar{n}/(\rho_2 q+\alpha_1)]^{\alpha_1\alpha_2}(\beta\alpha_0\alpha_2)^{\alpha_2-1}\mu^{\alpha_3}k_t^{\alpha_2(\alpha_2-1)}A_t^{\alpha_2\alpha_3}\varepsilon_t^{\alpha_2-1}exp\,(\alpha_3\xi_{t+1})\varepsilon_{t+1} \\ &= \Phi_t\Psi_{t+1}, \end{aligned} \tag{20}$$

because $A_{t+1} = A_t\;\mu\; exp(\xi_{t+1})$. $\Psi_{t+1} = exp\,(\alpha_3\xi_{t+1})\varepsilon_{t+1}$ and Φ_t denote all other right-hand side terms known in time t. Note that the conditional mean and variance of Ψ_{t+1} are time-invariant because ξ_{t+1} and ε_{t+1} are uncorrelated shocks independent of other variables. Since n_{t+1} is constant and k_t depends on production in time $t-1$, this equation actually informs us that the time series of stock returns are determined by the time series of technology shocks and non-technology shocks.[14]

Let g_{t+1} denote the economic growth ($g_{t+1} = F_{t+1}(\cdot)/F_t(\cdot)$), which is also the consumption growth and dividend growth. The expected stock returns are $E_t[R^s_{t+1}] = \Phi_t E_t[\Psi_{t+1}]$, which is proportional to expected consumption growth $E_t[R^s_{t+1}] \propto E_t[g_{t+1}]$. The relationship between expected stock returns and technology shocks can be derived as

$$\frac{\partial E_t[R^s_{t+1}]}{\partial\xi_t} = \frac{\partial\Phi_t}{\partial\xi_t}E_t[\Psi_{t+1}] = \alpha_2\alpha_3\Phi_t E_t[\Psi_{t+1}] = \frac{\alpha_2\alpha_3}{\beta}E_t[g_{t+1}] > 0. \tag{21}$$

because (1) $A_t^{\alpha_2\alpha_3} = A_{t-1}^{\alpha_2\alpha_3}\mu^{\alpha_2\alpha_3}\; exp(\alpha_2\alpha_3\xi_t)$ and $\partial A_t^{\alpha_2\alpha_3}/\partial\xi_t = \alpha_2\alpha_3 A_t^{\alpha_2\alpha_3} > 0$; (2) $\varepsilon_{t+1} \gg 0$ and $exp\,(\alpha_3\xi_{t+1})\varepsilon_{t+1} > 0$. As a result, a positive technology shock shall lead to higher expected market return in the next period.

[14] It can be derived that $k_t = const\; A_{t-1}^{\alpha_3}A_{t-2}^{\alpha_3\alpha_2}A_{t-3}^{\alpha_3\alpha_2^2}\ldots\varepsilon_{t-1}\varepsilon_{t-2}^{\alpha_2}\varepsilon_{t-3}^{\alpha_2^2}\ldots$, where $const$ denotes a constant.

A main reason of market premium is that market dislikes economic uncertainties that cause the stock return fluctuations. Here I derive the relationship between the expected market premiums, $E_t[R^s_{t+1} - R^f_{t+1}]$, and economic uncertainties. From $E_t[m_{t+1}R^s_{t+1}] = 1$,

$$\begin{aligned} E_t[R^s_{t+1} - R^f_{t+1}] &= \frac{-cov_t[m_{t+1}, R^s_{t+1}]}{E_t[m_{t+1}]} \\ &= \frac{-cov_t[(R^s_{t+1})^{-1}, R^s_{t+1}]}{E_t[(R^s_{t+1})^{-1}]} = \frac{-cov[\Psi^{-1}_{t+1}, \Psi_{t+1}]}{E[\Psi^{-1}_{t+1}]}\Phi_t \\ &= -corr[\Psi^{-1}_{t+1}, \Psi_{t+1}]SD(\Psi^{-1}_{t+1})SD(\Psi_{t+1})\frac{1}{E[\Psi^{-1}_{t+1}]}\Phi_t \\ &= -corr[\Psi^{-1}_{t+1}, \Psi_{t+1}]SD(\Psi^{-1}_{t+1})\frac{1}{E[\Psi^{-1}_{t+1}]}\frac{1}{\beta}SD_t(g_{t+1}) > 0, \end{aligned} \tag{22}$$

because $-corr[\Psi^{-1}_{t+1}, \Psi_{t+1}] > 0$. SD denotes standard deviation and $SD_t(g_{t+1}) = \beta\, SD_t(R^s_{t+1})$ $= \beta\Phi_t SD(\Psi_{t+1})$. This equation delivers two implications: first, the expected market premiums are always positive. This is very intuitive because that the technology shock and non-technology shock are two risk sources in this economy, and the agent must be compensated with corresponding risk premiums in holding the stock. Second, this equation illustrates that the time-variant market premium can be linked to time-variant volatility of economic growth and dividend growth. Since economic growth equals consumption growth in this model, this implication is consistent with the proposition of Bansal and Yaron (2004). In fact, the expected premium is proportional to conditional volatility of economic growth ($E_t[R^s_{t+1} - R^f_{t+1}] \propto SD_t(g_{t+1})$) because all other terms on the right hand side of Equation (22) are constants.

$$\frac{\partial E_t[R^s_{t+1}]}{\partial \xi_t} = \frac{\alpha_2\alpha_3}{\beta}E_t[g_{t+1}] > 0 \tag{23}$$

$$\frac{\partial E_t[R^s_{t+1} - R^f_{t+1}]}{\partial \xi_t} = -corr[\Psi^{-1}_{t+1}, \Psi_{t+1}]SD(\Psi^{-1}_{t+1})\frac{1}{E[\Psi^{-1}_{t+1}]}\frac{\alpha_2\alpha_3}{\beta}SD_t(g_{t+1}) > 0 \tag{24}$$

$$E_t[R^s_{t+1}] = \frac{1}{\beta}E_t[g_{t+1}] \tag{25}$$

$$E_t[R^s_{t+1} - R^f_{t+1}] = -corr[\Psi^{-1}_{t+1}, \Psi_{t+1}]SD(\Psi^{-1}_{t+1})\frac{1}{E[\Psi^{-1}_{t+1}]}\frac{1}{\beta}SD_t(g_{t+1}) \tag{26}$$

Then, the relationship between expected market premiums and technology shocks can be derived as

$$\frac{\partial E_t[R^s_{t+1} - R^f_{t+1}]}{\partial \xi_t} = -corr[\Psi^{-1}_{t+1}, \Psi_{t+1}]SD(\Psi^{-1}_{t+1})\frac{1}{E[\Psi^{-1}_{t+1}]}\frac{\alpha_2\alpha_3}{\beta}SD_t(g_{t+1}) > 0. \tag{27}$$

As a result, a positive technology shock shall lead to higher expected market premium in the next period. Here I exemplify the effect of a technology shock on conditional variance of economic growth.

(EXAMPLE 2) All settings in Example 1 apply here, except the non-technology shock process,

$$\{\varepsilon_t\}_{t=1}^{t^*+1} = \{\underbrace{1, 1, 1, ..., 1}_{t=1,...,t^*}, \varepsilon_{t^*+1}\},$$

and the variance of non-technology shock is σ_ε^2. The economic growth rates in time t^* and t^*+1 are

$$F_{t^*}(\cdot)/F_{t^*-1}(\cdot) = (1.5)^{\alpha_3}\varepsilon_{t^*}$$
$$F_{t^*+1}(\cdot)/F_{t^*}(\cdot) = (1.5)^{\alpha_2\alpha_3}\varepsilon_{t^*+1},$$

and their conditional variances are

$$Var_t[F_{t^*}(\cdot)/F_{t^*-1}(\cdot)] = (1.5)^{2\alpha_3}\sigma_\varepsilon^2$$
$$Var_t[F_{t^*+1}(\cdot)/F_{t^*}(\cdot)] = (1.5)^{2\alpha_2\alpha_3}\sigma_\varepsilon^2.$$

For comparison, I consider a benchmark case in which no technology shock exists, i.e. $\{A_t\}_{t=1}^{t^*+1} \equiv 1$. The conditional variance of consumption growth in this case is simply σ_ε^2. It is clear that $Var_t[F_{t^*+1}(\cdot)/F_{t^*}(\cdot)] > \sigma_\varepsilon^2$. So, it can be observed that a positive technology shock raises the uncertainty of economic growth, which results in higher market premium. The same as Example 1, the capital k plays a key role in the intertemporal causality between the uncorrelated technology shock and persistent variance of economic growth.

Some remarks regarding the implied predictability shown in Equations (21) and (27) are worth mentioning here. First, the magnitude of return predictability can be constant using log-linearization, which is $E_t[Ln(R_{t+1}^s)]/\partial\xi_t = \alpha_2\alpha_3$.[15] Second, all results derived in this paper are based on serially uncorrelated technology shocks. However, I can also use autocorrelated technology shocks in this model and deliver similar predictability. Although the magnitude of return and premium predictability may alter, but the their signs will remain positive. Finally, under current settings, this predictability is decreasing in time.[16] So, technology shocks' predictive power shall appear in short-term predictive regressions and may, but not necessarily, appear in long-term predictive regression.

3.4 Economic intuitions

Here I provide the the economic intuitions behind the return predictability and premium ways. The return predictability can be explained in several approaches. The first is based on the

[15] By (1) letting $r_{t+1}^s = ln(R_{t+1}^s)$; (2) log-linearizing Equation (20); and (3) differentiating that with respect to technology shock ξ_t, I find that: $\partial E_t[r_{t+1}^s]/\partial\xi_t = \alpha_2\alpha_3 > 0$. It is because that $ln(A_t) = \sum_{\tau=0}^{t} ln(\gamma_\tau)$, $\gamma_\tau = \mu\, exp\,(\xi_t)$, and ξ_t is independent of k_t, ξ_{t-1}, ε_t, and ε_{t+1}.

[16] For example, if I log-linearize the stock returns (r_{t+1}^s), then the h-step ahead predictability, $\partial E_t[r_{t+1}^s]/\partial\xi_{t-h} = {\alpha_2}^h\alpha_3$, shall diminish as h increases because $\alpha_2 < 1$ in general.

consumer's intertemporal substitution. By observing a positive technology shock occurring in this period, the agent knows that the productivity increases permanently. The permanent upshift in the production function due to a positive technology shock is analogous to an increase in the agent's permanent income (Friedman, 1957). As a result, the budget-constrained agent becomes more impatient and wants to consume more in this period. The agent also asks higher expected asset returns in the next period in exchange for this period's consumption. Such a relationship can be simply summarized by the relationship $m_{t+1} = (R^s_{t+1})^{-1} = \beta F_t(\cdot)/F_{t+1}(\cdot)$ (the inverse of economic growth). Second, in the model, the stock returns are determined by the dividend growth, which is exactly the economic growth. Since a positive technology shock in this period causes economic growth persisting for several periods, it therefore forecasts higher stock returns.

I can also explain the return predictability from the firm's perspectives. By observing a positive technology shock occurring in time t, the firm realizes that, given fixed labor input, the firm's investment return (precisely, the marginal product of capital) in time $t+1$ becomes higher.[17] Since the expected market return equals the expected investment return, it can also be predicted by the technology shock.

It is tempting to assert that, by observing a positive technology shock, the agent would react by buying more stocks and hence pushing its price higher. This argument, however, presumes a fixed stochastic discount rate and unlimited endowment or borrowing, and both are not true in this economy. By linking $m_{t+1} = (R^s_{t+1})^{-1}$ and Equation (20) together, we know that a positive technology shock in time t makes the agent prefer to consume more in time t and require higher expected asset returns due to lower m_{t+1}. The stochastic discount effects exactly offset the cashflow effects, and imply no contradiction to the market efficiency hypothesis.

Regarding the premium predictability, I start with a simple scenario. When many unexpected patents occur in this period, the agent expects more possible outcomes of production in the next period because all these patents are growth opportunities. Conceptually, the range of production growth rates becomes wider. The production growth is equal to economic growth, consumption growth, and dividend growth. So, the volatility of economic growth becomes higher and the agent should ask higher risk premium in holding market portfolio. This is consistent with Bansal and Yaron's (2004) model implications that the stock market dislikes economic uncertainty and conditional consumption volatility drives up market premium. Merton's (1973) intertemporal capital asset pricing model (ICAPM) provides another intuition. As a key component in agent's investment opportunity set, technology shocks can be treated as systematic risk factor. Based on all these reasonings, I propose that greater technology shocks lead to higher market premiums.

[17] The budget-constrained firm will increase its investment k_{t+1}, which decreases the marginal product of capital in time $t+1$ to some extent. However, the effect of a positive technology shock in time t will not be totally offset by the investment increase.

A relevant question is who provides the liquidity? I can assume a very small market maker in this economy who buys the stock in a positive technology shock and sells the stock in a negative technology shock. This small market maker can exist because the technology shocks have zero means.

In summary, a positive technology shock in this period increases both conditional mean and volatility of economic growth in the next period ($E_t[g_{t+1}]$ and $SD_t(g_{t+1})$). So, the next period' expected market return and premium should be higher as well.

4 Empirical Study

In the previous section, I find that, within the model contexts, the aggregate technology shocks predict future expected stock returns and premiums. In this section, I examine whether these model implications are supported by real data in both the U.S. and U.K.

4.1 Technology proxies and other data

I employ the total number of U.S. patents and total R&D expenditures to measure the aggregate technology level. Then, I compute the growth rates of U.S. patents and total R&D expenditures to gauge the aggregate technological growth, and detrend both growth rates to get proxies of the aggregate technology shock. As previous studies on macroeconomic predictors, I use quarterly data in the empirical study.[18] Here I briefly describe all data, and leave all details to Appendix B.

For the U.S. patent numbers, I use the U.S. patent applications data since 1976 that may be manually downloaded from the online database U.S. Patent Full-Text and Image Database (PatFT) of the U.S. Patent and Trademark Office (USPTO). As noted in Pakes (1985), these patent applications are "successful" patent applications since they are granted by USPTO sometime after being filed. Note that the successful patent application numbers are the only patent database available before March 2001, and has been widely used in the literature of industry organization. Following Pakes (1985), I presume the effective dates of these issued patent applications in U.S. are their application dates. I use the successful patent application number by the end of time t to measure the technology level in time t.[19] To measure the technological growth, I need to have a base of the total number of all patents filed before 1976. I estimate the number

[18] If I use annual data, the valid data of technology proxies can trace back only to the early 60s, which leaves us about forty sample points only. Another reason for quarterly data is to accommodate the lead time between technological inventions and production changes.

[19] I recognize that Abel's (1984) comments on Pakes (1985) state this is a strong assumption. However, I argue that some technology insiders, for example the patent law firms, can collect all qualified applications data to approximate the successful patent applications upon their filings and before they are eventually granted.

based on Hall, Jaffe, and Trajtenberg's (2001) dataset; since 1836, the total number of successful applications for U.S. patents amounts to be $4,065,811$ ($= A_0^{pat}$) at the end of 1975. The time series of the number of total patents is illustrated in the upper panel of Figure 1.

Some issues about using patents to measure technology are worth mentioning. First, although the level of patent applications may not match the aggregate technology level, their growth rates are presumed to be in a proportional relationship. Second, despite the heterogenous effects of patents, I use the argument of the "law of large numbers" (see Scherer, 1965; Griliches, 1990), and presume that all patent numbers are random variables from one distribution. Summing them up gives us the mean effect of all patents. Third, I recognize that, as a commonly used proxy, the number of successful patent applications may not be observed immediately because that there exists an about two year lag between application and issuance. However, according to USPTO's statistics, the ratio of patent issuances to patent applications is found to be stationary around 60% in my sample period (1976–2002).[20] So, the number of successful patent applications can be estimated using a constant ratio in the end of each period immediately.

Finally, I recognize one concern addressed by Jaffe and Lerner (2004) regarding the recent development of U.S. patent system. They report that the USPTO had approved many more patents since the early 1980s. I argue that the change in the U.S. patent system will not affect the validity of my empirical study for two reasons. First, the abnormal uptrend of patent growth, if any, will be removed in computing technology shocks because of which that I detrend the technological growth (illustrated in the lower panel of Figure 1) to compute technology shocks. I show that the time series of technology shocks is stationary as shown in Figure 2. Second, I consider another proxy (R&D expenses) and international evidence, both of which are immune to that patent anomaly.

For U.S. R&D expenditures, I sum up all quarterly R&D expenses (in millions of dollars) reported in the Compustat database and transform the number into 1996 dollars. A basis for the cumulative R&D expenditures to the end of 1988 is necessary to compute technological growth: I sum up the annual U.S. R&D expenditures in 1953-1988 reported in *National Patterns of Research and Development Resources:2003* of the National Science Foundation (2005, NSF hereafter) and obtain $3,299$ billion 1996 dollars ($= A_0^{rd}$) as the base level. Then, I add the quarterly total Compustat industry R&D expenditures to that base level and get an approximate accumulative industry R&D expenditures. This approximation is reasonable because that, according to the National Science Foundation (2005), industry R&D weighs 71.9% of total U.S. R&D expenses during 1990-2000. The growth rate of this approximate accumulative industry R&D expenditures is named US R&D growth hereafter.[21] I check this constructed quarterly R&D growth with the

[20]The first data file in the following linkage: http://www.uspto.gov/web/offices/ac/ido/oeip/taf/reports.htm

[21]I assume that the accumulative industry R&D expenditures obtained from Compustat are steadily proportional to accumulative aggregate R&D expenditures.

NSF's national annual R&D growth and find that they move consistently. The time series of R&D expenditures are illustrated in the upper panel of Figure 1.

Three issues about using R&D expenditures to measure technology are worth mentioning. First, although the input (R&D) may not fully become the output (technology), I hypothesize that the input-output ratio between R&D expenditures and the aggregate technology level is a constant. Second, I assume that the reported R&D expenses are optimal choices of firm managers following Bound et al. (1984), Pakes (1985), and others. Finally, it is well known that there exists a lag between the R&D input and technology output. Pakes and Schankerman (1984b) estimate that the mean lag is between 1.2 and 2.5 years, while this lag probably has become shorter in recent decades. I will accommodate this lag in constructing the proxy for the R&D-based technology shocks.

In the upper panel of Figure 1, I plot both the numbers of total patents in 1976Q1–2004Q3 and cumulative real R&D expenses in 1989Q1–2004Q3. It can be observed that these two series are smooth. In the lower panel of Figure 1, I plot the U.S. patent growth (r_t^{pat}) and R&D growth (r_t^{rd}) that are defined as follows:

$$r_t^{pat} = \frac{\text{Deseasonalized total patents by the end of time } t}{\text{Deseasonalized total patents by the end of time } t-1}$$
$$r_t^{rd} = \frac{\text{Deseasonalized cumulative real R\&D expenses by the end of time } t}{\text{Deseasonalized cumulative real R\&D expenses by the end of time } t-1},$$

where the deseasonalization method is a one-sided ratio to moving average-multiplicative method,[22] which does not to use future information in deseasonalization. It is clear that these two growth series show significant co-movement. Both series increase in the 90s, reach their peaks around 1998, and then start to decline after that. The drop of R&D growth can be attributed to the dropping off of internet companies after the bubble burst.

I then construct two proxies for the technology shocks based on r_t^{pat} and r_t^{rd}. The lower panel of Figure 1 demonstrates that both r_t^{pat} and r_t^{rd} contain stochastic trends. I employ a moving average detrending approach to disentangle the conditional expected growth and the unexpected shock.[23] I construct the first proxy for technology shocks based on patent growth ("patent shocks" hereafter) as follows:

$$\xi_t^{pat} = ln(r_{t-1}^{pat}) - \frac{1}{H}\sum_{h=1}^{H} ln(r_{t-1-h}^{pat}).$$

[22]For series $\{y_t\}$ with seasonality, I first compute the moving average $x_t = (y_t + y_{t-1} + y_{t-2})/3$. Then let $r_t = y_t/x_t$, and compute $s_t = r_t/\sqrt[4]{r_t r_{t-1} r_{t-2} r_{t-3}}$. Finally, the seasonally adjusted y_t, $y_t^* = y_t/s_t$.

[23]This fitting is motivated by Campbell's fitting for relative risk-free rate (1990, 1991), which is also a smooth time series with stochastic trends. Moreover, this setting avoids subjective model selection and forward-looking bias.

The second proxy is constructed based on R&D growth ("R&D shocks" hereafter) as follows:

$$\xi_t^{rd} = ln(r_{t-1}^{rd}) - \frac{1}{H}\sum_{h=1}^{H} ln(r_{t-1-h}^{rd}).$$

In constructing these two shock series, I have to impose one lag because that the technology shocks are assumed to occur in the beginning of each period in my model. It can be also regarded as a reporting lag between the inventions/discoveries and adoptions of new technologies.[24] The patent shock series ξ_t^{pat} and the R&D shock series ξ_t^{rd} are plotted in Figure 2. Due to the quarterly data frequency, I consider H to be four and eight. Moreover, I also considered other detrending methods including first-order difference as well as rolling AR(1). Due to space limitation, I reported the shocks based on moving average with $H = 4$ only in the context. Both shock series present stationarity without a significant trend, so the potential bias of an abnormal uptrend in patent growth proposed by Jaffe and Lerner (2004) no longer exists. The ξ_t^{pat} and ξ_t^{rd} are autocorrelated as real business cycle literature (e.g. King, Plosser, and Rebelo, 1988).[25] Moreover, these two shocks' sample means are close to zero, and show time-variant volatility.[26]

I use working hours as the labor input, real capital per capita as the capital input, and real GDP per capita as the production output. For the risk-free rate, I use the one-month Treasury Bill return. Several predictive variables are also considered in forecasting U.S. stock returns. They are "*cay*" of Lettau and Ludvigson (2001), the labor income to consumption ratio "SW" of Santos and Veronesi (2006), the dividend-price ratio "$d-p$" (Shiller, 1984; Campbell and Shiller, 1988; Fama and French, 1988), the dividend-earnings ratio "$d-e$" ("payout ratio", Lamont, 1998), the term spread "Term" and default premium "Default" (Fama and French, 1989), and the relative riskless rate "RRel" (Campbell, 1991). Complete description of all U.S. data is provided in Appendix B.

In Table 1, I report all summary statistics of variables used in this study, and some correlations between the technology shocks and other variables. Note that some variables are in logs while others are not. Numbers reported are consistent with recent studies (e.g. Lettau and Ludvigson, 2001; Santos and Veronesi, 2006). In the rightmost column of Panel A, I present the Augmented Dickey-Fuller (ADF) statistics (Dickey and Fuller, 1979; Said and Dickey, 1984) for all variables with first-order autocorrelation equal or larger than 0.85, and report corresponding critical values according to MacKinnon (1991).[27] It is found that all other predictors are very persistent except

[24] I recognize that a one quarter lag is a simplistic setting. However, it is reasonable to expect that aggregate R&D input starts to affect aggregate technology level two periods after.

[25] A potential argument is if the agent perceives the autocorrelation *ex ante*. Shephard and Harvey (1990) show that in finite samples, it is difficult to differentiate a process including an autocorrelated component from an i.i.d. process.

[26] The time-dependent volatility strengthens the premium predictability implied by the model.

[27] The lag number of models in computing ADF statistics are decided the model residuals' serial correlation, which should be zero. That is identified by Durbin-Watson statistics.

RRel, and only *cay* and the term spread reject the null hypothesis of the existence of a unit root according to the ADF test. The predictability results based on autocorrelated predictors call for advanced robustness checks. In the correlation reported in Panel B of Table 1, I report the following: (1) ξ^{pat} and ξ^{rd} are not highly correlated with other predictors; (2) ξ^{pat} and ξ^{rd} are positively correlated with CRSP returns and S&P 500 returns. The correlation between patent growth and R&D growth is 0.48, and the correlation between ξ^{pat} and ξ^{rd} is 0.12.

4.2 Predictive regressions

In Section 3.3, I have demonstrated that the expected market returns and premiums should be positively correlated with current technology shocks. In the empirical work, I regress the realized simple returns and excess returns on proxies of lagged technology shocks, $\{\xi_t\}_{t=1}^T$, and expect to obtain positive coefficients with significance.[28] I conduct unconditional regressions to estimate the average effect of technology shocks on market returns and premiums. I use the simple and logarithmic CRSP and S&P500 index returns and inflation-adjusted returns as proxies of market portfolio returns. Since I get almost identical results in all return measures, I only report the results of logarithmic CRSP inflation-adjusted returns, which is referred to as "CRSP index returns" hereafter. Also, I obtain similar results in different window sizes ($H = 1, 4, 8$) in detrending, hence I report only the $H = 4$ case in context. Moreover, I compare the explanatory/predictive power of technology shocks vis-a-vis other predictors at both short-term and long-term horizons.

In Table 2, I demonstrate that the patent shocks (ξ^{pat}) have significant predictive power for one-step ahead CRSP index returns. I consider both standardized patent shocks and original patent shocks as the predictors and find that, as the only regressor, their coefficients are all of significance. The t-statistics of regressions 1 and 8 are 3.37 and 3.38, respectively. The adjusted R^2s of regressions 1 and 8 are 0.05, which indicate that patent shocks explain 5% of total variance of (realized) total stock market returns.

Then, I run pairwise horseraces in a multivariate regression framework to compare the predictive abilities of patent shocks and other predictive variables. Most predictor candidates proposed in previous studies are considered in the horseraces: lagged returns (momentum), consumption to wealth ratio (*cay*), labor income to consumption ratio (SW), relative short-term rate (RRel), log dividend to price ratio ($d - p$), log dividend to earnings ratio ($d - e$), term spread (Term) and default spread (Default). In all other regressions in Table 2, I find that the patent shocks' coefficients are of significance and have higher t-statistics than other predictors.[29] Note that,

[28] Under rational expectations, the expected return should equal realized return in mean. I also recognize Elton's (1999) study indicating that the realized returns on average may not be an appropriate proxy for expected returns.

[29] Results of Term and Default are both insignificant and unreported due to the space limit.

the results of these pairwise horseraces do not imply that patent shocks outperform other predictors. Instead, I interpret these results as evidence for the claim that the predictive power from technology shocks are distinct from other macroeconomic or financial ratio predictors'. The insignificance of other predictors could be attributed to one of the following reasons: some change in the economy's structure happens during 1976–2004 (e.g. the shift of mean in states mentioned in Lettau and Van Nieuwerburgh, 2005); the 1-quarter horizon is not long enough for those predictors to perform; their effects disappear in a relatively volatile period, e.g. the 90s; some predictors, for example the labor income to consumption ratio (SW), are used to predict market premiums (excess returns) instead of market returns.

In Table 3, I report the results based on another proxy, the R&D shocks ξ^{rd}, which also shows significant predictive power for one-step ahead CRSP index returns. As with patent shocks, R&D shocks' coefficients in univariate predictive regressions are all of significance. The t-statistics of regressions 1 and 8 are 2.02 and 2.03, respectively. The adjusted R^2s of regressions 1 and 8 are 0.04, which indicate that the R&D shocks explain 4% of total variance of (realized) stock returns. In the pairwise horseraces, I also find that the R&D shock's coefficients are of significance (except regressions 5 and 12), while no other predictors have significant predictive ability. As a result, the model-implied market return predictability receives empirical support from patent and R&D data in the United States.

It is known that, in terms of predictability, the economic significance is as important as statistical significance. As reported in Tables 2 and 3, the coefficients of standardized ξ^{pat} and standardized ξ^{rd} are 0.02 and 0.03, respectively. Thus, a one standard deviation positive shock in patent shocks (R&D shocks) in this quarter implies a 2% (3%) increase in the expected market return in the next quarter. Similar numbers can be obtained by original patent and R&D shocks multiplied by their standard deviations. Note that, in regression 5 of Table 2, a one standard deviation decrease in RRel implies a 1.15% increase in the expected market returns in the next quarter ($-3.49 \times 0.326\%$). So, it is fair to state that the effect of technology shock on stock returns is of economic significance and reasonable magnitude.

In Figure 3, I exemplify technology shocks' predictive ability by plotting the realized CRSP index returns and the forecasted returns (the first regressions in Tables 2 and 3). It is observed that the technology shocks, especially in the 90s, can capture the trend of market returns. Not surprisingly, the forecasted return series is quite smooth because it aims to track the expected market returns, not the realized market returns.

I then consider the predictability in longer horizons by regressing the cumulative future market returns on technology shock proxies. Specifically, I consider Hodrick's (1992) 1B standard errors to account for the overlapping errors existing in the cumulative returns in order to draw a more correct inference.[30] As reported in Table 4, the patent shocks and R&D shocks maintain their

[30] Ang and Bekaert (2006) study the long-term return predictability and conclude that the performance of

predictive power throughout 4-, 8-, and 12-quarter horizons and produces commensurate adjusted R^2. It is also found that the intercept terms are consistently positive with significance.

Now, I consider the technology shock's predictive power for the market premium based on predictive regression.[31] For the market premium, I consider inflation-adjusted CRSP excess returns and logarithmic excess returns, inflation-adjusted S&P 500 excess returns and logarithmic excess returns. Since similar results are found in four cases, I report only the result of the inflation-adjusted CRSP logarithmic excess returns case (CRSP excess returns hereafter). In Table 5, I demonstrate that both standardized patent shocks and R&D shocks provide significant predictive power for one-step ahead CRSP excess returns. The adjusted R^2s of regressions 1 and 8 are 0.05 and 0.03, which indicate that the patent shocks (R&D shocks) explain 5% (3%) of total variance of (realized) excess returns. So, technology shocks' predictive power is of economic significance, and coefficients reported here are very close to those in Table 2 for market returns. I then consider the long-term performance of these two predictors. As reported in Table 6, the patent shocks and R&D shocks maintain their predictive power throughout 4-, 8-, and 12-quarter horizons and produces reasonable adjusted R^2. I note that, unlike what is found in market return predictability, the intercept terms are all insignificant.

Now, I consider the technology shock's predictive power for the real risk-free asset returns, which are one month T-bill returns from Ibbotson Associates minus the inflations in this study. In Panel A, I find that the patent shocks' predictive power for risk-free asset returns is marginal with t-value 1.61 in the sample period 1977Q1–2004Q3. In Panel B and C based on the 90s sample periods, I find that both patent shocks and R&D shocks do produce significantly positive coefficients for risk-free asset returns and provide high adjusted R^2. One possible explanation for this mixed outcome is that we usually use the short-term rate as the proxy for risk-free rate, and the short-term rate rate is basically controlled by the Fed instead of a outcome of market equilibrium (in the short-term). The Fed sets the short-term rate based on a combination of different short- and long-term policy targets, and the Fed's paradigm of monetary policies is time-variant.

Based on all these findings, I conclude that the technology shock proxies, ξ^{pat} and ξ^{rd}, can explain the changes in expected future market returns and premiums. Using horserace regressions and pairwise correlations, I further show that technology shocks have predictive power in explaining a distinct part of market return variation that has been explained by other macroeconomic variables and financial ratios. Most importantly, this return predictability found in the empirical study is consistent with theoretical modeling and cannot be simply attributed to data snooping. As an additional note, the predictability outcome simply describes the movement of

Hodrick's 1B standard errors is much better than the Newey-West (1987) standard errors or the robust GMM generalization of Hansen and Hodrick (1980).

[31] I recognized that the magnitude of correlation implied in my model is in fact time variant.

“fair” expected asset returns associated with technology shocks, and it should not be interpreted as a claim of practical profitability based on technology-related information. It is simply because that the income effect is totally offset by the substitution effect in the log utility function. Some “real-time” issues against profitability, therefore, are less relevant to this study.

Another noteworthy finding in this section is that not all other predictors have significant predictive power for returns or premiums in the examined sample period. This phenomenon is not uncommon in the literature (e.g., Goyal and Welch, 2006) and may be explained by the following: (1) a fundamental change occurred in the U.S. economy during 1976–2004; (2) other predictors' predictive power is diluted in the volatile period, e.g. the 90s. Goyal and Welch (2006) found that most predictors perform poorly over the last 30 years; (3) the quarterly data frequency considered in this study may be inappropriate for some predictors, e.g. the labor income to consumption ratio.

4.3 Robustness checks

4.3.1 Issues in data

Perhaps the most intriguing question is whether the predictability based on technology shocks is a special consequence of the internet boom and burst period. To answer this question, I examine their predictability in the subsample 1977Q1-1995Q4 and report the results in Table 8. It can be observed that the patent shocks, ξ^{pat}, still significantly predict CRSP index and excess returns, and the coefficients and adjusted R^2 are close to the whole sample results. Another interesting in this table is that cay, RRel, and $d-p$ present strong predictive power. A possible explanations for this finding is that these indicators' predictive power is based on earlier sample period (e.g. Goyal and Welch, 2006). It would be ideal if we could check this predictability with data prior to 1970. However, due to the unavoidable limitation in data availability, researchers cannot obtain appropriate technology proxies before 1970 (e.g. Chan, Lakonishok, and Sougiannis, 2001; Rossi, 2005). Finally, even if this predictability occurs only after the 1980s, it does not eliminate my model since a regime change that new technologies change the aggregate production more effective and faster in these decades.

Out-of-sample issue is another reasonable concern. In the earlier version of this paper, I used the sample period ends in 2002Q4 and obtained similar return predictability and premium predictability.

Equations (21) and (27) indicate that the magnitude of predictability is time-variant. So, I conduct rolling regressions to inspect the time-variant relationship between technology shocks and expected market returns as well as premiums. I regress the realized simple returns and excess returns on lagged patent shocks with a rolling window of 80 quarters. In Table 9, I found that the the coefficients of standardized patent shocks maintain significantly positive across different sample periods. It is also interesting to find that the lowest t-statistics occur in 1981Q1 – 2000Q4. This may even strengthen my conclusion because the predictability is not weakened if more data points are included in the sample.

I also check the contemporaneous relationship between market returns and technology shocks. I regress the S&P 500 returns on concurrent patent shocks and R&D shocks, respectively, and obtain significantly positive coefficients. As a result, the contemporaneous relationship between market returns and technology shocks is verified.

I recognize that the "reporting lag" (Balvers, Cosimano, and McDonald, 1990) is an important issue in predictability. Since most macroeconomic and financial ratio predictors are not immediately available at the end of a period, the relevant information is not fully revealed to the market (even insiders) in the beginning of the next period. To take this concern into account, I impose one more lag and run two-step ahead predictive regressions in patent shocks case.[32] In Table 10, I obtain results very close to Table 2. In R&D shocks case, the results in Table 3 have included one quarter for the input-output lag. Therefore, accommodating this possible reporting lag does not alter the conclusion that the technology shocks forecast market returns.

4.3.2 Issues in econometrics

An important econometric issue in predictive regression is that the coefficients affiliated to autocorrelated predictors are upward-biased, especially in the small sample (e.g. Stambaugh, 1986, 1999).[33] In Table 11, I report the bias based on Stambaugh's (1999) estimation and its effect on coefficient estimates. It is found that, although the patent shocks and R&D shocks are autocorrelated, their innovations (residuals of AR(1) model) do not correlate with the predictive regression's residuals (c_1 are not significant in Panel C). As a result, the possible small sample bias is negligible and does not alter the conclusion of predictability.

Employing bootstrap-based tests is another way to examine if the predictive power is merely a small sample phenomenon. In my implementation, the test statistics are the coefficients affiliated with standardized patent and R&D shocks. I use the simple bootstrap to build the null distribution (no predictability) of test statistics and find that the bootstrap p-values of regression 1 in Tables 2 and 3 are 0.007 and 0.031, respectively. The bootstrap test therefore confirms the predictability. The details of my implementation are as follows. I first regress the market returns on a constant and technology shocks, and get the residual series. Second, I randomly resample the residual series to form the resampled residual series. By adding this resampled residual series to the estimated constant, I construct the resampled market return series, and then I regress this

[32] I argue that one quarter is long enough for the market, or at least insiders (e.g. large patent law firms or USPTO staff), to adjust the stock prices according to the technology shocks.

[33] On the other hand, Lewellen (2004) and Cochrane (2006) both comment that Stambaugh's estimation may substantially understate the predictability in short-term forecasting.

resampled market return series on technology shocks to estimate $\hat{\beta}^b$ (this process is repeated for $1,000$ times, i.e. $b = 1, ..., 1,000$). Finally, by comparing the test statistic to the distribution of $\{\hat{\beta}^b\}_{b=1,...,1,000}$, I obtain the bootstrap p-value for testing.

Another common problem with small samples is the existence influential point. It is possible that the significant coefficients in predictive regressions are caused by some influential data points. To check this point, I plot the stock returns and lagged technology shocks and find a clear trend line. Therefore, the found predictability is not induced by influential points.

Boudoukh, Richardson, and Whitelaw (2005) address one concern about long-term predictability based on autocorrelated predictors: The multi-horizon predictive regressions are almost perfectly correlated with short-term predictive regressions. Their concern is less relevant to this empirical study because: First, my theoretical model, stated as Equations (21) and (27), mainly states short-term predictability. Therefore, instead of claiming long-term predictability, I regard the significant results found in long-term predictive regressions as supportive evidence for my short-term predictability. Second, both U.S. patent shocks and R&D shocks are less autocorrelated than most other predictors, so it is less likely that their long-term predictability is simply a replication of their short-term predictability.

4.3.3 GMM estimation for the system

Last, I conduct the generalized method of moments (GMM) estimation and J-test (Hansen, 1982) to check if the whole model is misspecified. The closed-form solution derived in Section 3.2 and 3.3 can be empirically tested. The details of GMM estimation and J-test are provided in Appendix D. I use the data of output and investment in 1977Q1–2004Q3 and conduct the standard two-step procedure GMM estimation to estimate the mean and standard deviations of parameters. The J-test statistic and Newey-West's (1987) covariance matrix estimate are implemented. In Table 12, I find that all free parameters are of significance in lag 4 and 8: the estimated subjective discount factor β are 0.97 and 0.97, which are significantly lower than one; estimated α_1 (for labor) are 0.69 and 0.67, estimated α_2 (for capital) are 0.40 and 0.40, and estimated α_3 (for technology) are 0.42 and 0.41 – all are significantly larger than zero. The p-values of the J-test are 0.93 in Newey-West lag 4 case and 0.96 in Newey-West lag 8 case, which imply that all moment conditions do not significantly deviate from zeros. As a result, the GMM estimation indicates that the proposed model is properly specified, and most importantly the model-impled return predictability exists (because α_2 and α_3 are confirmed to be significantly positive). However, I recognize that, if I use consumption data instead of investment data, the model is strongly rejected by J-test as widely perceived in the literature.

4.4 International evidence: U.K.

In this section, I inspect: (1) whether British patent data help to explain the pattern of real U.K. output better; and (2) whether U.K. technology shocks measured from patent data has explanatory power for expected U.K. stock returns.

I manually collect the British patent applications from the *Patents and Designs Journal* published weekly by the Patent Office of the United Kingdom.[34] I can therefore construct the time series of British patent growth and U.K. technology shocks using a procedure similar to that used in U.S. patent case. By the end of 1989, the total number of applications filed for British patent since 1948 amounted to $1,878,250$ according to the data provided by the U.K. Patent Office. Thus I can compute the British patent growth r_t^{UKpat} and British technology shocks ξ_t^{UKpat} using a procedure identical to that used in U.S. data. For the market returns, I use the FTSE 100 index returns provided by Yahoo!Finance.[35] In Figure 4, I plot both the British patent application growth and the British patent shocks in 1991Q1–2004Q4. It is reasonable to conclude that the British patent shock series represents a stationary time series. The descriptions of all other macro-variables including the inflation, labor input (working hour), real capital input per capita, and real output (GDP) per capita are left to Appendix C.

First, to examine the relationship between technological growth and real output growth, I regress the logarithmic growth of real GDP per worker on logarithmic growth rates of labor hours, real investment per worker, and technology. As mentioned, I use the British patent growth rates to measure technological growth. In Table 13, I find that the real GDP per capita can be better explained with U.K. technological growth. The regression in Panel A delivers adjusted R^2 0.189, while the regression in Panel B delivers adjusted R^2 0.081 only. Meanwhile, the coefficient of technological growth in Panel A has statistical significance. I thus confirm that British patent data is a reasonable proxy for technology in general and better explains U.K. output dynamics.

In Table 14, I run short-term and long-term predictive regression to examine whether British technology shocks predict future FTSE100 index returns in logs. In the short-term predictability reported in Panel A, I find that standardized British patent shocks predict future FTSE100 index returns with significance, while the lagged FTSE100 index returns do not carry any predictive power. The coefficients of ξ^{UKpat} are 0.021 and 0.023 in regressions 1 and 3, and these numbers are close to the coefficients found in the U.S. data. In Figure 5, I plot the actual returns and the fitted returns based on regression 1 of Table 14. It is observed that the predicted expected stock return matches the trend of realized stock returns and, not surprisingly, the predicted return series is less volatile than the realized one, because I propose the predictability in expected market returns.

[34]Note that the number is total British patent applications, not successful patent applications. Nevertheless, this is the only available measure of U.K. patent accumulations by quarter.

[35]http://finance.yahoo.com

Finally, I inspect the predictability at longer horizons and report the results in Panel B of Table 14. I regress the cumulative future FTSE100 index returns in logs on the British patent shock, and find no significant predictability in 4-, 8-, and 12-quarter horizons according to Hodrick 1B method (1992). I therefore conclude that, contrary to the findings in U.S. data, patent shocks do not carry predictive power for long-term U.K. stock returns.

5 A General Model and Its Numerical Solution

5.1 The model and Bellman equation

For the purpose of generality, I also consider a more general model (Model 2 henceforth) with an inseparable power utility function

$$u(c_t, \bar{n} - n_t) = (1/\delta)[c_t^{\eta}(\bar{n} - n_t)^{1-\eta}]^{\delta}, \tag{28}$$

and capital accumulation. This utility function has been considered by Kydland and Prescott (1982) and Danthine, Donaldson and Johnson (1998). To enable the calibration, I use two conditions that are common in the literature: (1) the risk-free asset is in zero net supply in all periods, i.e. $\{b_t\}_{t=1}^{T} \equiv 0$; (2) the total shares of the stock is normalized to one in all periods, i.e. $\{s_t\}_{t=1}^{\infty} \equiv 1$,[36] and transfer the agent's problem into a Pareto optimum problem:

$$\begin{aligned} &\max_{k_{t+1}, n_t} E_t \left[\sum_{\tau=1}^{\infty} \beta^{\tau} (1/\delta)[c_{t+\tau}^{\eta}(\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta} \right] \\ &s.t. \quad c_t + k_{t+1} = \alpha_0 {n_t}^{\alpha_1} k_t^{\alpha_2} {A_t}^{\alpha_3} \varepsilon_t + (1 - \Omega) k_t, \end{aligned} \tag{29}$$

where Ω is the depreciation rate.[37] It can be observed that the optimal investment and labor policies are mainly decided by the agent, which is intuitive because the agent is the shareholder of the firm and guides the firm to conduct the resource allocation.My numerical solution to Model 2

relies on the recursive value function iteration technique of Christiano (1990a, 1990b), which has been used by Danthine, Donaldson, and Mehra (1989, DDM hereafter) and Danthine, Donaldson, and Johnsen (1998, DDJ hereafter) to search for the optimal policy in a real business cycle study. The first task is to deflate consumption and capital for stationarity like DDJ. One way is to set $\alpha_1 = \alpha_3$ and $\alpha_2 + \alpha_3 = 1$ (constant return to scale), and have the agent's maximization problem become:

$$\begin{aligned} &\max_{\hat{k}_{t+1}, n_t} E_t \left[{\gamma_t}^{\eta\delta}(1/\delta)[c_t^{\eta}(\bar{n} - n_t)^{1-\eta}]^{\delta} + \sum_{\tau=1}^{\infty} (\prod_{s=1}^{t} \beta {\gamma_s}^{\eta\delta})(1/\delta)[c_t^{\eta}(\bar{n} - n_t)^{1-\eta}]^{\delta} \right] \\ &s.t. \quad \hat{c}_t + \hat{k}_{t+1}\gamma_{t+1} = \alpha_0 {n_t}^{\alpha_1} \hat{k}_t^{\alpha_2} \varepsilon_t + (1 - \Omega)\hat{k}_t, \end{aligned} \tag{30}$$

[36] These two conditions do not affect the first order conditions in solving the n_t and k_{t+1}.

[37] There is no adjustment cost in this model.

where $\hat{c}_t = c_t/A_t$ and $\hat{k}_t = k_t/A_t$.[38] I then apply the recursive value function iteration technique to identify optimal policies (choice variables), $\hat{k}_{t+1}$ and n_t, under different states (i.e. various combinations of state variables $\hat{k}_t$, γ_t, and ε_t). In each iteration i (time is fixed), the decision variables are $\hat{k}_i$ and n_i, and the state variables in each period include $\hat{k}$, γ, and ε. All state variables are assigned discrete numbers. All feasible capital choice variable, $\hat{k}$, lie in a domain $S_k \equiv \{\hat{k}^1, \hat{k}^2, ..., \hat{k}^{lk}\}$, where $\hat{k}^1 < \hat{k}^2 < ... < \hat{k}^{lk}$ and $\hat{k}^{i+1} - \hat{k}^i = \hat{k}^i - \hat{k}^{i-1}$, and lk denotes the total number of capital choices. To calibrate the model, I simplify the technological growth γ as three values: γ_1, γ_2, and γ_3 with transition probability $\{p^{\xi}_{i,j}\}_{i,j=1,2,3}$. The non-technology shock $\varepsilon = \{\overline{\varepsilon}, \underline{\varepsilon}\}$ with probability $\{1/2, 1/2\}$.

The sequence of approximating value functions is illustrated by the following Bellman equation:

$$\begin{aligned} V_i(\hat{k}, \gamma, \varepsilon) = \max_{\{\hat{k}_i, n_i\} \in \Gamma} \{ & \frac{1}{\delta}[(\alpha_0 {n_i}^{\alpha_1} \hat{k}^{\alpha_2} \varepsilon + (1-\Omega)\hat{k} - \hat{k}_i)^{\eta} (\bar{n} - n_i)^{1-\eta}]^{\delta} \\ & + \beta \sum_{h=1}^{3} \sum_{j=1}^{2} \gamma_h^{\eta\delta} V_{i-1}(\hat{k}_i, \gamma_h, \varepsilon_j) p^{\xi}_h \frac{1}{2} \}, \end{aligned} \tag{31}$$

subject to

$$0 \le \hat{k}_i \le \alpha_0 {n_i}^{\alpha_1} \hat{k}^{\alpha_2} \varepsilon + (1-\Omega)\hat{k},$$
$$0 \le n_i \le \bar{n}.$$

V_i denotes the i-th iteration with current state variables $\hat{k}$, γ, and ε. h is the next state of γ, p^{ξ}_h denotes the transition probability between current state and next state of γ, and j is the next state of ε. Γ is the domain of feasible choice variables $\hat{k}_i$ and n_i ($\Gamma = \{S_k \times [0, \bar{n}]\}$).

In searching for optimal policy, DMM note that the second term on the right hand side of equation (31) is irrelevant to choice of n_i. So, I can maximize the utility $u(\cdot)$ with respect to n_i first to find the optimal n_i by fixed point approximation.[39] Then, I can employ an exhaustive search to find the optimal $\hat{k}_i$ in S_k. The details of optimal policy search is described in Appendix E.

5.2 Calibration and return predictability

I calibrate the model at the quarterly frequency and set $\beta = 0.98$, $\Omega = 0.05$, $\bar{n} = 1$, initial labor $n_0 = 0.3$, and initial capital $k_0 = 0.5$. There are 201 total possible capital levels (grid points) evenly distributed on the range between the minimum $k^1 = 0.01$ and maximum $k^{201} = 2.01$. The

[38] $\max E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} (1/\delta) [c^{\eta}_{t+\tau} (\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta}\right] = \max E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} (1/\delta) [c^{\eta}_{t+\tau} (\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta} (A_{t+\tau}/A_{t+\tau})^{\eta\delta}\right]$ $= \max_{\hat{k}_{t+1}, n_t} E_t \left[\sum_{\tau=0}^{\infty} \beta^{\tau} (1/\delta) [\hat{c}^{\eta}_{t+\tau} (\bar{n} - n_{t+\tau})^{1-\eta}]^{\delta} (A_{t+\tau})^{\eta\delta}\right]$. Note $A_t = (\prod_{s=1}^{t} \gamma_s) A_0$ and all A_0 terms can be neglected.

[39] The optimal n_t, as shown in Equation (49) in Appendix E, may be analytically solved instead.

upper bound and lower bound are set so as to avoid the optimal choice occurring close to them. For the agent's utility function, I set $\eta = 0.333$ and $\delta = -0.1$ following Kydland and Prescott (1982). For the production function, I set $\alpha_0 = 15$, $\alpha_1 = 0.64$, $\alpha_2 = 0.36$, and $\alpha_3 = 0.64$. The value of α_0 is chosen to deliver: (1) output growth that approximates the historical data; and (2) positive dividend series. For technology growth, three possible values of γ are $[1, 1.005, 1.01]$ with probability $p^{\xi}_{i,i} = 0.5$ and $p^{\xi}_{i,j} = 0.25$ $(i \neq j)$. The technology shock is defined as $\gamma_t - \gamma_{t-1}$ in this procedure. For non-technology shocks, I set $\overline{\varepsilon} = 1.005$ with probability 0.5 and $\underline{\varepsilon} = 0.995$ with probability 0.5.

I randomly simulate $\{\varepsilon_t\}_{t=1}^T$ and $\{\gamma_t\}_{t=1}^T$ to initiate the dynamics. The state variable set in each period t, $\{\hat{k}_t, \gamma_t, \varepsilon_t\}$, can be computed according to these shocks and initial conditions described in the preceding paragraph. Then, the optimal policy set $\{\hat{k}_{t+1}, n_t\}_{t=1}^T$ can be identified with the searching procedure described in Appendix E.

By adjusting for the deflator A_t, I obtain the output $\{F(n_t, k_t, A_t, \varepsilon_t)\}_{t=1}^T$, consumption $\{c_t\}_{t=1}^T$, and capital $\{k_t\}_{t=1}^T$. The time series of wages and the dividends, $\{w_t\}_{t=1}^T$ and $\{d_t\}_{t=1}^T$, are decided by the firm's decision: The equilibrium wage equals the marginal product of labor, and the dividend equals $F(n_t, k_t, A_t, \varepsilon_t) - k_{t+1} + (1 - \Omega)k_t - n_t w_t$.

Given the pricing kernel,

$$m_t = \beta \frac{\partial u(c_t, \bar{n} - n_t)/\partial c_t}{\partial u(c_{t-1}, \bar{n} - n_{t-1})/\partial c_{t-1}}, \tag{32}$$

I can derive the risk-free asset returns, $\{R_t^f\}_{t=1}^T$, as follows: $R_t^f = 1/E_{t-1}[m_t]$.

I assume that the expected stock price equals to the present value of total discounted dividends in 200 sequential periods:

$$p_t = \sum_{\tau=1}^{200} \left[\left(\prod_{i=t+1}^{t+\tau} m_i \right) d_{t+\tau} \right]. \tag{33}$$

So, I can derive realized returns $\{R_t^s\}_{t=1}^T$ according to $R_t^s = (p_t + d_t)/p_{t-1}$.[40]

Then, I generate ten simulations, and each simulation is of length 1,000. I use all the variables in the period $t = 101$ to $t = 700$ to compute their means and standard errors. The averages of means and standard errors of ten simulations are reported in Table 15. I first compare the calibrated economic dynamics with the historical data: As mentioned, my target is to match the historical output growth. The calibrated mean output growth and standard deviation are 0.005 and 0.008, both approximate the historical data. The calibrated mean capital growth, consumption growth, and productivity growth are all 0.005, which are slightly lower than historical data. The calibrated mean risk-free asset return is greater than the historical data (0.013 vs.

[40] Due to the difficulty of computing expected returns, I use this simplistic and *ex post* setting. Nevertheless, it is a reasonable setting because realized returns should be equal to expected returns on average.

0.006), while the calibrated mean stock return is smaller than the historical value (0.015 vs. 0.022). So, the market premium generated in this model is 0.002 per quarter.

The most important task here is to check the technology shock's effect in this economy. I report the correlation between technology shocks and output growth, next period's stock returns, and next period's excess returns. As shown in Table 15, the technology shock is highly correlated with output growth (88%), which verifies the fact that technology shock is the main driver of this economy. Then, I find that the stock returns and premiums in next period are positively correlated with current technology shocks (5% and 0.6%). This outcome is consistent with the previous simple model implication and empirical findings. Therefore, the return predictability and premium predictability exist not only in the simple log-economy but also in a more general economy.

6 Conclusions

This paper highlights the role of technology shocks in market returns and contributes to the finance literature by providing an analytical model and new empirical evidence. From the theoretical perspective, I construct an economy that solves the dynamics of consumption, production, labor market, and financial assets in a general equilibrium framework. My model characterizes an aggregate technology component in the production function, in which I demonstrate that technology shocks affect expected market returns and premiums across time. This model is novel because it provides an exact closed-form solution rather than the conventional log-linear approximation. The main mechanism can be interpreted as follows: technology shocks drive up the means and volatilities of economic growth, and therefore imply time-variant market returns and premiums.

Perhaps the most important contribution of this paper is its empirical results due to the scarcity of empirical evidence in the literature (e.g. Lettau, 2003; Panageas and Yu, 2006). In this paper, I find that U.S. technology shocks, measured by unexpected growth rates in U.S. patents and R&D expenses, significantly explain future raw and excess returns of the CRSP index and the S&P500 index at different horizons. More interestingly, they outperform other predictors including the consumption to wealth ratio, labor income to consumption, relative risk-free rate, dividend to price ratio, payout ratio, default premium, and term spread. This finding survives several robustness checks. As a result, technology shocks can be measured and utilized to explain a distinct portion of market returns and premiums in time series. Moreover, I also find that the U.K. patent shocks forecast the FTSE100 index and excess returns. These findings justify the direction of technology shocks' influence on market returns and premiums. My empirical study therefore suggests the potential of technology-relevant variables in asset pricing research.

Appendices

A. The social planer's problem

Because the unique Pareto optimal allocation proposed by a central planner model must coincide with the outcome of a competitive equilibrium model, I can solve the planning version of the model described in Section 3.2 and derive the equilibrium consumption and labor policy functions. They will then be imposed into the decentralized version of the model in Section 3.2, which allows me to solve all other variables.

I assume there is a single, infinitely lived representative agent (consumer-worker-investor) whose problem is to maximize her/his time-additive expected utility in time t as follows

$$\max_{c_t, n_t} \{u(c_t, \bar{n} - n_t) + \sum_{\tau=1}^{\infty} \beta^{\tau} E_t \left[u(c_{t+\tau}, \bar{n} - n_{t+\tau}) \right] \} \tag{34}$$

$$\begin{aligned} s.t. \;\; & c_t = F_t(n_t, k_t, A_t, \epsilon_t) - k_{t+1}, \\ & F(n_t, k_t, A_t, \epsilon_t) = \alpha_0 {n_t}^{\alpha_1} k_t^{\alpha_2} {A_t}^{\alpha_3} \varepsilon_t \\ & A_t = A_{t-1} \gamma_t, \quad \gamma_t = \mu exp\,(\xi_t), \end{aligned}$$

where β is a subjective discount rate ($0 < \beta < 1$), and $u(c_t, \bar{n}_t - n_t)$ characterizes the agent's period utility function that depends on the agent's consumption c_t and leisure $\bar{n} - n_t$ in time t; n_t denotes the labor input and $\bar{n}$ denotes total available time. k_{t+1} is the investment reserved for capital stock in the next period, which fully depreciates in production in time $t+1$. The firm's production function, $F(n_t, k_t, A_t, \epsilon_t)$, follows a Cobb-Douglas form that contains labor input n_t, capital input k_t, a technology component A_t, and a non-technology production shock ε_t. For notational simplicity, I use $F_t(\cdot)$ instead of $F(n_t, k_t, A_t, \epsilon_t)$ hereafter. I assume that $0 < \alpha_1, \alpha_2, \alpha_3 < 1$. A_t denotes the technology level at time t that is the compound of technological growth, γ_t, since time 0. Since technological growth is persistent across time, I assume it follows a logarithmic random walk process with mean μ and an unexpected permanent technology shock in growth, ξ, which satisfies $E_{t-1}[exp\,(\xi_t)] = 1$. ξ is distributed with mean ν_ξ and variance σ_ξ^2, and $exp\,(\xi_t) \geq 1/\mu$. The last term in the production function, ε_t, represents the unexpected temporary non-technology shock in level that is i.i.d. and satisfies $E_{t-1}[\varepsilon_t] = 1$ and $E_{t-1}[ln(\varepsilon_t)] = \nu_\varepsilon$, which accommodates all other uncertainties and is independent of the technology shock and other contemporaneous variables. There is only one good in this economy, and it is perishable so that the agent can only consume it today or invest it for tomorrow's production. In each time t, the agent first observes the technology shock and non-technology shock, and then decides the consumption, investment, and working time based on expectations for the future.

The agent's utility function is assumed to follow

$$u(c_t, \bar{n} - n_t) = \rho_1 \, ln(c_t) + \rho_2 \, ln(\bar{n} - n_t), \tag{35}$$

where ρ_1 and ρ_2 are strictly positive. The value function corresponding to the problem in Equation (34) is as follows:

$$V(k_t, A_{t-1}, \xi_t, \varepsilon_t) = \max_{c_t, n_t}\{\rho_1 ln(c_t) + \rho_2 ln(\bar{n} - n_t) + \beta E_t[V(k_{t+1}, A_t, \xi_{t+1}, \varepsilon_{t+1})]\}. \quad (36)$$

Note that: (1) The state variable arguments of $V(\cdot)$ represent all information known to the agent in making her/his labor, consumption, and investment allocation decisions: capital stock, preceding technology level, current technological shock, and current non-technology shock; (2) I set $c_t = q_t F_t(\cdot)$, where q_t is the fraction of output to be consumed in time t, and therefore $k_{t+1} = (1 - q_t)F_t(\cdot)$.

I conjecture the solution is of the following value function form:

$$V(k_t, A_{t-1}, \xi_t, \varepsilon_t) = \phi_1 + \phi_2 ln(k_t) + \phi_3 ln(A_{t-1}) + \phi_4 \xi_t + \phi_5 ln(\varepsilon_t), \quad (37)$$

which is solvable and has an explicit unique solution.[41] By taking the expectation of Equation (37), we know that

$$\begin{aligned} E_t[V(k_{t+1}, A_t, \xi_{t+1}, \varepsilon_{t+1})] &= E_t\{\phi_1 + \phi_2\, ln(k_{t+1}) + \phi_3\, ln(A_t) + \phi_4\, \xi_{t+1} + \phi_5\, ln(\varepsilon_{t+1})\} \\ &= \phi_1 + \phi_2\, ln((1 - q_t)F_t(\cdot)) + \phi_3\, ln(A_t) + \phi_4 E_t[\xi_{t+1}] + \phi_5 E_t[ln(\varepsilon_{t+1})] \\ &= \phi_1 + \phi_2\, ln(1 - q_t) + \phi_2\, ln(F_t(\cdot)) + \phi_3\, ln(A_t) + \phi_4\, \nu_\xi + \phi_5\, \nu_\varepsilon, \end{aligned}$$

because $ln(\gamma_{t+1}) = ln(\mu) + \xi_{t+1}$, $E_t[\xi_{t+1}] = \nu_\xi$, and $E_t[ln(\varepsilon_{t+1})] = \nu_\varepsilon$.

I then rewrite the maximization problem in Equation (36) as:

$$V(k_t, A_{t-1}, \xi_t, \varepsilon_t) = \max_{q_t, n_t}\{\rho_1 ln(q_t F_t(\cdot)) + \rho_2 ln(\bar{n} - n_t) + \beta E_t[V(k_{t+1}, A_t, \xi_{t+1}, \varepsilon_{t+1})]\}, \quad (38)$$

which can be derived as:

$$\begin{aligned} V(k_t, A_{t-1}, \xi_t, \varepsilon_t) = \max_{q_t, n_t}\{&\rho_1 ln(q_t) + \rho_1 ln(F_t(\cdot)) + \rho_2 ln(\bar{n} - n_t) \\ &+\beta\left[\phi_1 + \phi_2\, ln(1 - q_t) + \phi_2\, ln(F_t(\cdot)) + \phi_3\, ln(A_t) + \phi_4\, \nu_\xi + \phi_5\, \nu_\varepsilon\right]\}. \quad (39) \end{aligned}$$

Since the value function on the right hand side is concave with respect to n_t and q_t, I can use the FOCs to find the maximum. Because $ln(F_t(\cdot)) = ln(\alpha_0) + \alpha_1 ln(n_t) + \alpha_2 ln(k_t) + \alpha_3 ln(A_t) + ln(\varepsilon_t)$, the FOC with respect to n_t is

$$\begin{aligned} 0 &= \rho_1 \frac{\partial\, ln(F_t(\cdot))}{\partial\, n_t} - \frac{\rho_2}{\bar{n} - n_t} + \beta\phi_2 \frac{\partial\, ln(F_t(\cdot))}{\partial n_t} \\ &= \frac{\rho_1 \alpha_1}{n_t} - \frac{\rho_2}{\bar{n} - n_t} + \frac{\beta\phi_2\alpha_1}{n_t} \\ \text{So,}\quad n_t &= \frac{\alpha_1(\rho_1 + \beta\phi_2)\bar{n}}{\rho_2 + \alpha_1(\rho_1 + \beta\phi_2)}. \end{aligned}$$

[41] I benefited from a discussion with Jack Favilukis in solving this equation.

Also, the FOC with respect to q_t is

$$0 = \frac{\rho_1}{q_t} - \frac{\beta\phi_2}{1-q_t}.$$
$$\text{So,} \quad q_t = \frac{\rho_1}{\rho_1 + \beta\phi_2}.$$

Taking form (37), n_t, and q_t into Equation (39), I can solve for other parameters:

$$\phi_1 + \phi_2 ln(k_t) + \phi_3 ln(A_{t-1}) + \phi_4 \xi_t + \phi_5 ln(\varepsilon_t) = \rho_1 ln(q_t) + \rho_1 ln(F_t(\cdot)) + \rho_2 ln(\bar{n} - n_t)$$
$$+\beta\left[\phi_1 + \phi_2\, ln(1-q_t) + \phi_2\, ln(F_t(\cdot)) + \phi_3\, ln(A_t) + \phi_4\, \nu_\xi + \phi_5\, \nu_\varepsilon\right]. \quad (40)$$

The right hand side can be further expanded by decomposing $ln(F_t(\cdot)) = ln(\alpha_0) + \alpha_1 ln(n_t) + \alpha_2 ln(k_t) + \alpha_3 ln(A_t) + ln(\varepsilon_t)$:

$$\rho_1 ln(q_t) + \rho_1 ln(\alpha_0) + \rho_1\alpha_1 ln(n_t) + \rho_1\alpha_2 ln(k_t) + \rho_1\alpha_3 ln(A_t) + \rho_1 ln(\varepsilon_t) + \rho_2 ln(\bar{n} - n_t)$$
$$+\beta\phi_1 + \beta\phi_2 ln(1-q_t) + \beta\phi_2 ln(\alpha_0) + \beta\phi_2\alpha_1 ln(n_t) + \beta\phi_2\alpha_2 ln(k_t) + \beta\phi_2\alpha_3 ln(A_t) + \beta\phi_2 ln(\varepsilon_t) + \beta\phi_3 ln(A_t)$$
$$+\beta\phi_3 ln(\mu) + \beta\phi_4\nu_\xi + \beta\phi_5\nu_\varepsilon \quad (41)$$

By matching the coefficient of each variable on the left hand side of Equation (40) to those in the right hand side shown in Equation (41), I can solve all parameters for this planner's version. I note that the change of $ln(k_t)$ is independent of other state variables, which implies that the coefficients on both sides must match:

$$\phi_2 = \rho_1\alpha_2 + \beta\phi_2\alpha_2,$$
$$\text{which implies} \quad \phi_2 = \frac{\rho_1\alpha_2}{1-\beta\alpha_2} > 0.$$

It is noted that ϕ_2 is a constant. Similarly, for $ln(\varepsilon_t)$,

$$\phi_5 = \rho_1 + \beta\phi_2 > 0.$$

Then, I decompose $ln(A_t)$ in Equation (41) because $ln(A_t) = ln(A_{t-1}) + ln(\mu) + \xi_t$. Since ξ_t and $ln(A_{t-1})$ are independent of other variables, I get the following results:

$$\phi_3 = \rho_1\alpha_3 + \beta\phi_2\alpha_3 + \beta\phi_3, \text{ and } \phi_4 = \phi_3$$
$$\text{which implies} \quad \phi_3 = \phi_4 = \frac{\rho_1\alpha_3 + \beta\phi_2\alpha_3}{1-\beta} > 0.$$

By taking the result that ϕ_2 is constant and strictly positive to the representation of n_t and q_t, I obtain constant solutions for n_t and q_t and use notation n and q for them hereafter. Therefore, all other terms on the right hand side are constants or ϕ_1-related, and the value ϕ_1 is solved as a constant. It is intuitive that all coefficients except the intercept are positive. It is also found that n and q lie in meaningful ranges: $\bar{n} > n > 0$ and $1 > q > 0$. Moreover, since all coefficients $\{\phi_j\}_{j=1,\ldots,5}$ are unique solutions, the $\{n, q\}$ are unique.

Therefore, I have demonstrated that the proposed value function $V(k_t, A_{t-1}, \xi_t, \varepsilon_t)$ is valid and provides a unique solution that satisfies general economic intuitions. Last, but not least, I derive two main results. First, $c_t = qF_t(\cdot)$ or equivalently, $k_{t+1} = (1-q)F_t(\cdot)$. This can be used directly in the decentralized economy in the context. Second, the labor input is constant ($n_t = \frac{\alpha_1(\rho_1+\beta\phi_2)\bar{n}}{\rho_2+\alpha_1(\rho_1+\beta\phi_2)}$), which will be verified in the decentralized economy.

Moreover, I can set the technological growth as an endogenous process by letting $\mu_t = \alpha_4 k_t^{\alpha_5}$ and still solve this problem in the same way. The only two required conditions are

$$\alpha_4 > 0$$
$$\alpha_5 < \frac{1-\beta\alpha_2}{\beta\alpha_3 + \beta^2\alpha_3/(1-\beta)},$$

and these two conditions are satisfied in the empirical data. In the solution of this setting, the equilibrium labor input and capital to output ration are still constants.

B. U.S. data in details

1. Gross domestic product (GDP) per capita: the real, seasonally adjusted GDP (ID: GDPC96) divided by population. Data is obtained from Federal Reserve Economic Data (FRED).[42] The unit is in billions of chained U.S. dollars in 2000.

2. Population: the total population including all ages and armed forces overseas (ID: POP) from FRED. Since the data is in monthly frequency, I use the three-month average as the quarterly population. The unit is in thousands.

3. Labor input: the average weekly work hours of production workers (ID: CES0500000005) divided by the hours of five days. Data is obtained from the Department of Labor, Bureau of Labor Statistics.[43] The unit is hours, and the series is seasonally adjusted. The data is in monthly frequency, and I use the three-month average number as the quarterly data.

4. Investment and capital per capita: the total investment per capita is the sum of real gross private domestic investment (ID: GPDIC96), real federal nondefense gross investment (ID: NDGIC96), and real state and local government gross investment (ID: SLINVC96) divided by the population. All data are from FRED. All data are seasonally adjusted. The unit is in billions of chained U.S. dollars in 2000. To compute the capital, I accumulate the total investment of each quarter since 1947Q1 with depreciation rate 2.5% per quarter.

[42]FRED: http://research.stlouisfed.org/fred2/

[43]Website of U.S. Department of Labor, Bureau of Labor Statistics: http://www.bls.gov/

5. Price and inflation: The price level used here is the consumer price index for all urban consumers including all items (ID: CPIAUCSL). It is seasonal adjusted (monthly) and its base period is 1982-84 (= 100). I use the three-month average of the price index as the price level of that quarter. The data is obtained from FRED, and the original source is the Department of Labor, Bureau of Labor Statistics.

6. Market returns: I consider the CRSP value-weighted index returns and S&P500 index returns as two proxies for market portfolio returns. I also consider inflation-adjusted returns, for which I adjust market returns with inflation measured by the growth rate of the price level. The CRSP value-weighted returns come from Kenneth French's website.[44] S&P500 index returns series come from CRSP Monthly Stock dataset, which does not include the dividends.

7. Risk-free asset returns: One month treasury-bill returns from Ibbotson Associates are also available from Kenneth French's website.

8. U.S. patent applications: I find some inappropriate data points (1982Q2-Q3 and 1995Q1-Q2) that appear unreasonable jumps, so I substitute them using an interpolation method. Moreover, because there exists a lag between the application date and granted date of each patent, the patent application number in the period 2002-2004 requires estimation. I multiply the number of filed applications reported by USPTO with an estimated granted ratio based on Published Applications Database. This estimation does not affect my conclusion because I obtain return predictability and premium predictability in sample periods 1976Q1–1995Q4 and 1976Q1–2001Q4.

9. U.S. R&D expenses: Some more issues to be addressed here: First, for firms that report only annual R&D expenses, I divide their annual expenses by four as their quarterly expenses. Nevertheless, I also construct alternative R&D growth and shocks based on quarterly reported R&D expenditures only and obtain similar return predictability and premium predictability. Second, since the total R&D expenses in 2004 are so low that I tend to consider them as outliers. By excluding 2004, however, I still find return predictability and premium predictability in the sample period 1991Q2–2003Q4.

10. *cay*: from Lettau and Ludvigson (2001).[45] I prolong the original *cay* data series to 2004Q3 based on the same formula ($cay = c - 0.3054a - 0.5891y$) with updated c, a, y available from the same source.

11. Labor income to consumption ratio (SW): the predictor SW is constructed following the calculation described in Santos and Veronesi (2006). They define the (aggregate) labor income as: compensation of employees, received (Line 2) (= wage and salary disbursements

[44]I thank Kenneth French for sharing the data. http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.
[45]I thank Martin Lettau for making *cay* data available via http://pages.stern.nyu.edu/ mlettau/.

+ supplements to wages and salaries) + personal current transfer receipts (Line 16) - contributions for government social insurance (line 24) - personal current taxes (line 25).[46] All items are in National Income and Product Accounts (NIPA) Table 2.1: Personal Income and Its Disposition. Data are from FRED database.

12. Relative riskfree rate (RRel): current one-month Treasury bill rate minus the previous 4-quarter average of that rate.

13. Dividend-price ratio: the dividend-price ratio series is obtained from Robert Shiller's website.[47] The ratio is based on S&P500 composite index. Since the dividend data is not available after June 2004, I assume that the dividends in September and December 2004 are at the same level as June 2004.

14. Dividend-earnings ratio: the dividend-earnings ratio is also obtained from Robert Shiller's website.

15. Term spread (Term): 10-year government bond rate (constant maturity) minus 3-month T-bill rate (secondary market), both from FRED.

16. Default premium (Default): Moody's BAA corporate bond rate minus AAA corporate bond rate, both from FRED.

C. U.K. data in details

Except patent data, all data are taken from the Office for National Statistics, U.K.[48]

1. British patent: I manually collect the quarterly number of all British patent applications reported in the *Patents and Designs Journal* published weekly by the Patent Office of the United Kingdom. In each issue, I record the page numbers and estimate the number of applications on each page to estimate the patent applications. The *Patents and Designs Journal* has not changed its version since Issue 5212, published in January 1989.

2. Gross domestic product (GDP) per capita: the real, seasonally adjusted GDP (ID: IHXW) divided by U.K. population. The unit is in millions of 2003 £.

3. Population: I use the people in employment of UK: aged 16 and older (ID: MGRN LFS) instead of the total population because the latter is available in yearly frequency only in my search. The series is seasonally adjusted, and the unit is in thousands.

[46] The consumption defined here is the personal consumption expenditures on nondurable goods and services (lines 6 and 13) in Table 2.3.5 of NIPA: Personal Consumption. Labor income to consumption ratio is the labor income divided by consumption in each period. Accruals are neglected in my study.

[47] I acknowledge Robert Shiller for making the data available via http://www.econ.yale.edu/ shiller/data.htm.

[48] http://www.statistics.gov.uk

4. Labor input: the average weekly work hours. I compute that number by dividing the total actual weekly hours worked (ID: YBUS LFS) (in millions) by population and hours of five days. Both series are seasonally adjusted.

5. Investment input per capita: I divide the real total gross fixed capital formation (ID: NPQT) by population. Both series are seasonally adjusted. The unit is in millions of 2002 £.

6. U.K. stock returns: the FTSE 100 index returns obtained from Yahoo!Finance.[49]

D. GMM estimation

Based on the closed-form solution derived in Section 3.2, I can propose an empirically testable joint hypothesis as follows:

$$
\begin{aligned}
\text{Freeparameters:} \quad & \beta,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ const_1 \\
\text{Production function:} \quad & 0 = E_t[\Delta F_{t+1} - (\Delta n_{t+1})^{\alpha_1}(\Delta k_{t+1})^{\alpha_2}\gamma_{t+1}^{\alpha_3}] && (42) \\
\text{Investment to output ratio:} \quad & 0 = E_t[k_{t+1} - \beta\alpha_2\, F_t] && (43) \\
\text{Stock and investment returns:} \quad & 0 = E_t[R^s_{t+1} - \beta^{-1}\Delta F_{t+1}] && (44) \\
\text{Euler equation:} \quad & 0 = E_t[m_{t+1}R^s_{t+1} - 1] && (45) \\
\text{Stock predictability:} \quad & 0 = E_t[r^s_{t+1} - const_1 - \alpha_2\alpha_3\xi_t] && (46)
\end{aligned}
$$

where the stochastic discount factor $m_{t+1} = \beta(\Delta n_{t+1})^{-\alpha_1}(\Delta k_{t+1})^{-\alpha_2}\gamma_{t+1}^{-\alpha_3}(\Delta\varepsilon_{t+1})^{-1}$, Δ denotes the gross growth, and $r^s = ln(R^s)$. β, α_1, α_2, α_3, q, and $const_1$ are free parameters (α_0 is not included because it is a scalar in the model). I try to make this system include all parameters and important economic variables that can be measured by available data, but also recognize possible limitations of this hypothesis setting and GMM estimation.[50]

I use real GDP per capita for F, the growth of real GDP per capita for ΔF, growth of labor hours for Δn, growth of real capital per capita for k, growth of real capital per capita for Δk, patent growth γ^{pat} for γ, patent shocks ξ^{pat} for ξ, logarithmic CRSP index and excess returns for r^s and r^e, and ε are estimated based on following equation:

$$ln(\Delta F(n_t, k_t, A_t, \varepsilon_t)) = \alpha_1 ln(\Delta n_t) + \alpha_2 ln(\Delta k_t) + \alpha_3 ln(\gamma_t) + \epsilon_t. \tag{47}$$

Note that the details of data are described in Appendix B.

Equations (42) to (46) are exactly the moment conditions I can impose in GMM estimation,

[49] http://finance.yahoo.com

[50] Limitations include: (1) I do not include the wage data due to the lack of an appropriate measure for real labor income; (2) I consider investment data instead of consumption data; and (3) I do not consider excess return and risk-free rate due to the lack of corresponding closed-form moment condition.

and their sample analogs are:

$$
\begin{aligned}
0 &= \frac{1}{T}\sum_{t=1}^{T}\left[\left(\Delta F_{t+1} - (\Delta n_{t+1})^{\alpha_1}(\Delta k_{t+1})^{\alpha_2}\gamma_{t+1}^{\alpha_3}\right) z_t\right] \\
0 &= \frac{1}{T}\sum_{t=1}^{T}\left[k_{t+1} - \beta\alpha_2\, F_t\right], \\
0 &= \frac{1}{T}\sum_{t=1}^{T}\left[\left(R_{t+1}^s - \beta^{-1}\Delta F_{t+1}\right) z_t\right], \\
0 &= \frac{1}{T}\sum_{t=1}^{T}\left[\left(m_{t+1}R_{t+1}^s - 1\right) z_t\right], \\
0 &= \frac{1}{T}\sum_{t=1}^{T}\left[\left(r_{t+1}^s - const_1 - \alpha_2\alpha_3\xi_t\right) z_t\right],
\end{aligned}
$$

where z_t denotes the instrumental variables. I use the constant and two-step-lagged time series of production growth, labor growth, investment growth, and index returns as instrumental variables (i.e. $z_t = [1, \Delta F_{t-1}, \Delta n_{t-1}, \Delta k_t, R_{t-1}^s]$) because of the existence of ξ_t. To account for autocorrelation and heteroskedasticity of time series data, I use the Newey-West's (1987) covariance matrix estimate with lag number $nw = 4$ and 8 (note that the generally recommended lag number is 4, $floor(T^{1/3}) = 4$).

E. Optimal policy search

Here I demonstrate the details in searching for optimal policy n_i and $\hat{k}_i$. As mentioned in the Bellman equation formation, I can solve the optimal n_i associated with each combination of state variables by maximizing the utility function:

$$
\begin{aligned}
u(\hat{c}, \bar{n} - n_i) &= \frac{1}{\delta}[\hat{c}^{\eta}(\bar{n} - n_i)^{1-\eta}]^{\delta} \\
&= \frac{1}{\delta}(\alpha_0 n_i^{\alpha_1}\hat{k}^{\alpha_2}\varepsilon - \hat{k}_i + (1-\Omega)\hat{k})^{\eta\delta}(\bar{n} - n_i)^{(1-\eta)\delta} \\
&= \frac{1}{\delta}(\mathcal{X} n_i^{\alpha_1} - \hat{k}_i + (1-\Omega)\hat{k})^{\eta\delta}(\bar{n} - n_i)^{(1-\eta)\delta}, \qquad (48) \\
\text{where} \quad & \mathcal{X} = \alpha_0\hat{k}^{\alpha_2}\varepsilon \geq 0.
\end{aligned}
$$

I note that equation (48) is a concave function of n_i, so I may differentiate the utility function with respect to n_i to find the maximum:

$$\begin{aligned}
0 &= \frac{\partial u(\hat{c}, \bar{n} - n_i)}{\partial n_i} \\
&= \frac{1}{\delta}\eta\delta(\mathcal{X}n_i^{\alpha_1} - \hat{k}_i + (1-\Omega)\hat{k})^{\eta\delta-1}(\alpha_1\mathcal{X}n_i^{\alpha_1-1})(\bar{n} - n_i)^{(1-\eta)\delta} \\
&\quad + \frac{1}{\delta}(1-\eta)\delta(\bar{n} - n_i)^{(1-\eta)\delta-1}(-1)(\mathcal{X}n_i^{\alpha_1} - \hat{k}_i + (1-\Omega)\hat{k})^{\eta\delta} \\
&= (\mathcal{X}n_i^{\alpha_1} - \hat{k}_i + (1-\Omega)\hat{k})^{\eta\delta-1}(\bar{n} - n_i)^{(1-\eta)\delta-1} \\
&\quad \{(\bar{n} - n_i)\eta\alpha_1\mathcal{X}n_i^{\alpha_1-1} - (1-\eta)(\mathcal{X}n_i^{\alpha_1} - \hat{k}_i + (1-\Omega)\hat{k})\}.
\end{aligned}$$

Since neither $\mathcal{X}n_i^{\alpha_1} - \hat{k}_i + (1-\Omega)\hat{k} = 0$ (no consumption) nor $\bar{n} - n_i = 0$ (no leisure) can be optimal for the agent, a solution to the equation above necessitates:

$$\begin{aligned}
(\bar{n} - n_i)\eta\alpha_1\mathcal{X}n_i^{\alpha_1-1} &= (1-\eta)(\mathcal{X}n_i^{\alpha_1} - \hat{k}_i - (1-\Omega)\hat{k}) \\
\bar{n}\eta\alpha_1\mathcal{X}n_i^{\alpha_1-1} - \eta\alpha_1\mathcal{X}n_i^{\alpha_1} &= (1-\eta)\mathcal{X}n_i^{\alpha_1} - (1-\eta)(\hat{k}_i - (1-\Omega)\hat{k}) \\
\bar{n}\eta\alpha_1\mathcal{X} + (1-\eta)(\hat{k}_i - (1-\Omega)\hat{k})n_i^{1-\alpha_1} &= (1-\eta+\eta\alpha_1)\mathcal{X}n_i,
\end{aligned}$$

which implies

$$\begin{aligned}
n_i &= \frac{\bar{n}\eta\alpha_1\mathcal{X}}{(1-\eta+\eta\alpha_1)\mathcal{X}} + \frac{(1-\eta)(\hat{k}_i - (1-\Omega)\hat{k})}{(1-\eta+\eta\alpha_1)\mathcal{X}}n_i^{1-\alpha_1} \\
&= \mathcal{B} + \mathcal{C}n_i^{1-\alpha_1}, \qquad (49)
\end{aligned}$$

$$\text{where} \quad \mathcal{B} = \frac{\bar{n}\eta\alpha_1\mathcal{X}}{(1-\eta+\eta\alpha_1)\mathcal{X}}, \;\; \mathcal{C} = \frac{(1-\eta)(\hat{k}_i - (1-\Omega)\hat{k})}{(1-\eta+\eta\alpha_1)\mathcal{X}}. \qquad (50)$$

Therefore, I can find the optimal n_i by using a fixed point procedure based on Equation (49). I then use the standard procedure (recursive iteration) in Danthine, Donaldson, and Mehra (1989) to approximate the optimal n_i. Given the known state variables and determined optimal labor n_i, I then use the exhaustive search to find the optimal k_i, which is the grid point in the set S_k that maximizes Equation (31).

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Table 1: Summary statistics

Panel A reports the descriptive statistics of all variables, and Panel B reports the contemporaneous correlation between technology shocks and stock returns and other predictors. The sample period for most variables is 1976Q1–2004Q3, except for patent shocks (1977Q1–2004Q3), R&D shocks (1991Q2–2004Q3), and British data (1991Q1–2004Q4). The t statistics reported are the results of testing whether the means of variables are different from zero. Selected Augmented Dickey-Fuller (ADF) statistics of some highly autocorrelated variables are reported in the last column, and the following $*$ denotes that ADF test (with intercept) rejects the existence of a unit root with 10% significance level. The lag number of models in computing ADF statistics are decided according to the model's Durbin-Watson statistic and the t-statistic of coefficients of lagged variable as regressor.

Panel A: Descriptive statistics

Variables	Mean (%)	Median (%)	Max. (%)	Min. (%)	Std. dev. (%)	t-stat. (zero)	1st order autocor.	ADF stat.
Asset returns								
CRSPvw	3.064	3.831	19.202	-25.998	8.183	4.033	-0.043	
S&P500	2.240	2.437	18.952	-26.432	7.794	3.095	-0.001	
r^f	1.494	1.342	3.737	0.220	0.746	21.575	0.949	-1.44
Inflation	0.895	0.694	2.770	0.195	0.576	16.723	0.905	-2.37 $*$
FTSE 100	1.465	1.393	16.156	-20.071	7.932	1.431	-0.003	
Technology-related variables								
r_t^{pat}	0.543	0.516	0.779	0.341	0.136	42.965	0.989	-0.47
r_t^{rd}	1.175	1.189	1.383	0.799	0.127	69.490	0.965	2.17
r_t^{UKpat}	0.459	0.362	0.951	0.309	0.163	162.599	0.997	-6.79 $*$
ξ_t^{pat}	0.006	0.005	0.085	-0.095	0.028	2.481	0.633	
ξ_t^{rd}	-0.011	0.000	0.089	-0.207	0.062	-1.269	0.820	
ξ_t^{UKpat}	-0.001	-0.002	0.046	-0.062	0.021	-0.445	0.463	
Other predictors								
cay	60.581	60.899	63.710	56.477	1.307		0.863	-2.86 $*$
SW	89.462	88.207	98.178	82.670	4.238		0.982	-1.96
RRel	-0.024	-0.021	0.811	-0.955	0.326		0.668	
$d-p$	-355.492	-345.123	-277.936	-449.807	48.153		0.989	-0.37
$d-e$	-77.683	-84.122	-26.928	-118.980	20.362		0.951	-1.20
Term	1.793	1.852	3.800	-1.430	1.267		0.861	-2.95 $*$
Default	1.084	0.983	2.513	0.560	0.432		0.912	-2.42

Panel B: Correlation between technology shocks and other variables

	CRSPvw	S&P500	cay	SW	RRel	$d-p$	$d-e$	Term	Default
ξ_t^{pat}	0.067	0.094	0.155	-0.161	0.179	-0.023	-0.114	0.145	-0.173
ξ_t^{rd}	0.134	0.191	-0.152	-0.107	0.263	0.215	0.048	-0.384	-0.388
	FTSE100								
ξ_t^{UKpat}	-0.198								

Table 2: Short-term forecasting for CRSP index returns: Patent shocks and others

I regress the log CRSP inflation-adjusted returns of time $t+1$ on patent shocks and other predictors in time t (1-step ahead forecasting): $r^s_{t+1} = X_t\beta + e_{t+1}$, where X_t denotes a vector of predictors, β denotes a vector of coefficients, and e_{t+1} denotes the residual. "Lag Ret" denotes the lagged CRSP inflation-adjusted returns, i.e. r^s_t. The descriptions of all other predictive variables can be found in the context or in Appendix B. Numbers in parentheses are the t-statistics of Newey-West's estimator (1987), adjusted for serial correlation and heteroskedasticity up to four lags. The sample period is 1977Q1–2004Q3. I use the standardized patent shocks as the predictor in Panel A, and the original patent shocks as the predictor in Panel B. Numbers in bold indicate p-values of t-statistics (one-sided) that are smaller than 5%.

Panel A: Standardized patent shocks and other predictors									
♯	Const.	Lag Ret	Stand. ξ^{pat}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
1	**0.02**		**0.02**						0.05
	(3.07)		(3.37)						
2	**0.02**	-0.05	**0.02**						0.04
	(3.03)	(-0.70)	(3.44)						
3	-0.40		**0.02**	0.69					0.05
	(-1.07)		(3.09)	(1.14)					
4	-0.08		**0.02**		0.12				0.04
	(-0.51)		(3.21)		(0.64)				
5	**0.02**		**0.02**			**-3.49**			0.06
	(3.05)		(3.55)			(-1.77)			
6	**0.10**		**0.02**				0.02		0.06
	(1.70)		(3.38)				(1.29)		
7	0.03		**0.02**					0.01	0.04
	(1.05)		(3.27)					(0.35)	
Panel B: Patent shocks and other predictors									
♯	Const.	Lag Ret	ξ^{pat}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
8	**0.02**		**72.83**						0.05
	(2.17)		(3.38)						
9	**0.02**	-0.05	**74.24**						0.04
	(2.16)	(-0.70)	(3.43)						
10	-0.40		**67.83**	0.69					0.05
	(-1.09)		(3.09)	(1.14)					
11	-0.09		**75.61**		0.12				0.04
	(-0.54)		(3.21)		(0.65)				
12	**0.02**		**80.42**			**-3.49**			0.06
	(2.03)		(3.55)			(-1.77)			
13	0.09		**73.76**				0.22		0.06
	(1.63)		(3.38)				(1.30)		
14	0.03		**73.96**					0.01	0.04
	(0.90)		(3.27)					(0.35)	

Table 3: Short-term forecasting for CRSP index returns: R&D shocks and others

I regress the log CRSP inflation-adjusted returns of time $t+1$ on R&D shocks and other predictors in time t (1-step ahead forecasting): $r^s_{t+1} = X_t\beta + e_{t+1}$, where X_t denotes a vector of predictors, β denotes a vector of coefficients, and e_{t+1} denotes the residual. "Lag Ret" denotes the lagged CRSP inflation-adjusted returns, i.e. r^s_t. The descriptions of all other predictive variables can be found in the context or in Appendix B. Numbers in parentheses are the t-statistics of Newey-West's estimator (1987), adjusted for serial correlation and heteroskedasticity up to four lags. The sample period is 1991Q2–2004Q3. I use the standardized R&D shocks as the predictor in Panel A, and the original R&D shocks as the predictor in Panel B. Numbers in bold indicate p-values of t-statistics (one-sided) that are smaller than 5%.

Panel A: Standardized R&D shocks and other predictors

♯	Const.	Lag Ret	Stand. ξ^{rd}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
1	0.02		**0.03**						0.04
	(1.25)		(2.02)						
2	0.02	-0.10	**0.03**						0.03
	(1.23)	(-0.87)	(2.08)						
3	-0.57		**0.03**	0.98					0.06
	(-1.52)		(1.97)	(1.58)					
4	-0.60		**0.03**		0.72				0.04
	(-0.88)		(2.03)		(0.91)				
5	0.02		0.02			2.88			0.03
	(1.21)		(1.59)			(0.52)			
6	**0.22**		**0.02**				0.05		0.06
	(1.68)		(1.83)				(1.51)		
7	0.03		**0.02**					0.01	0.04
	(1.05)		(3.27)					(0.18)	

Panel B: R&D shocks and other predictors

♯	Const.	Lag Ret	ξ^{rd}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
8	**0.02**		**43.88**						0.04
	(1.88)		(2.03)						
9	**0.02**	-0.10	**46.92**						0.03
	(1.80)	(-0.87)	(2.08)						
10	-0.57		**51.80**	0.98					0.06
	(-1.51)		(1.98)	(1.58)					
11	-0.60		**46.37**		0.72				0.05
	(-0.87)		(2.03)		(0.91)				
12	**0.02**		38.47			2.87			0.03
	(1.74)		(1.60)			(0.52)			
13	**0.22**		**38.64**				0.05		0.06
	(1.71)		(1.83)				(1.51)		
14	0.03		**44.92**					0.01	0.02
	(0.86)		(1.95)					(0.18)	

Table 4: Long-term forecasting for CRSP index returns: Patent shocks and R&D shocks

I use cumulative CRSP index return as stock market returns, and run the following long-term predictive regression: $r^s_{t+k} + ... + r^s_{t+1} = a_0 + a_1\, \xi_t + u_{t+k,t}$, where k denotes the length of forecasting horizon and $u_{t+k,t}$ denotes the overlapping residual. The sample sizes involving predictors ξ^{pat} and ξ^{rd} are 1977Q1–2004Q3 and 1991Q2–2004Q3, respectively. Numbers in brackets are t-statistics based on Hodrick 1B (1992) standard errors that are designed for cumulative predictive regression. Numbers in boldface indicate significance under 5% level (one-sided) with 1B standard errors.

Panel A: Future 4-Quarter Returns				
♯	const.	stand ξ^{pat}	stand ξ^{rd}	$adj - R^2$
1	**0.09**	**0.07**		0.18
	[2.73]	[2.66]		
2	**0.07**		**0.07**	0.07
	[2.06]		[2.51]	
Panel B: Future 8-Quarter Returns				
♯	const.	stand ξ^{pat}	stand ξ^{rd}	$adj - R^2$
3	**0.18**	**0.06**		0.08
	[2.71]	[1.74]		
4	**0.13**		**0.10**	0.04
	[1.97]		[2.51]	
Panel C: Future 12-Quarter Returns				
♯	const.	ξ^{pat}	ξ^{rd}	$adj - R^2$
5	**0.26**	**0.10**		0.10
	[2.66]	[2.40]		
6	**0.17**		**0.16**	0.05
	[1.69]		[3.24]	

Table 5: Short-term forecasting for CRSP excess returns: Patent shocks, R&D shocks, and others

I regress the logarithmic CRSP inflation-adjusted excess returns of time $t+1$ on standardized patent shocks (Panel A), standardized R&D shocks (Panel B), and other predictors in time t (1-step ahead forecasting): $r^s_{t+1} - r^f_{t+1} = X_t\beta + e_{t+1}$, where X_t denotes a vector of predictors, β denotes a vector of coefficients, and e_{t+1} denotes the residual. "Lag Ret" denotes the lagged CRSP excess returns, i.e. $r^s_t - r^f_t$. The descriptions of all other predictive variables can be found in the context or in Appendix B. The sample sizes involving predictors stand ξ^{pat} and stand ξ^{rd} are 1977Q1–2004Q3 and 1991Q2–2004Q3, respectively. Numbers in parentheses are the t-statistics of Newey-West's estimator (1987), adjusted for serial correlation and heteroskedasticity up to four lags. Numbers in bold indicate p-values of t-statistics (one-sided) that are smaller than 5%.

Panel A: Standardized patent shocks and other predictors									
♯	Const.	Lag Ret	Stand. ξ^{pat}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
1	0.01		**0.02**						0.05
	(0.85)		(3.53)						
2	0.01	-0.05	**0.02**						0.05
	(0.85)	(-0.62)	(3.61)						
3	-0.40		**0.02**	0.68					0.06
	(-1.05)		(3.29)	(1.07)					
4	0.01		**0.02**		0.00				0.04
	(0.03)		(3.23)		(0.00)				
5	0.00		**0.02**			**-4.51**			0.08
	(0.74)		(3.78)			(-2.49)			
6	0.04		**0.02**				0.01		0.05
	(0.72)		(3.47)				(0.61)		
7	0.02		**0.02**					0.02	0.05
	(0.86)		(3.40)					(0.64)	
Panel B: Standardized R&D shocks and other predictors									
♯	Const.	Lag Ret	ξ^{rd}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
8	0.01		**0.03**						0.03
	(0.51)		(1.86)						
9	0.01	-0.10	**0.03**						0.02
	(0.53)	(-0.81)	(1.90)						
10	-0.65		**0.03**	**1.10**					0.06
	(-1.68)		(1.85)	(1.72)					
11	-0.70		**0.03**		0.83				0.04
	(-1.02)		(1.88)		(1.03)				
12	0.01		0.02			2.16			0.02
	(0.55)		(1.47)			(0.38)			
13	0.21		**0.02**				0.05		0.05
	(1.65)		(1.66)				(1.54)		
14	0.02		**0.03**					0.01	0.02
	(0.54)		(1.79)					(0.30)	

Table 6: Long-term forecasting for CRSP excess returns: Patent shocks and R&D shocks

I use cumulative CRSP excess return as stock market returns, and run the following long-term predictive regression: $r^s_{t+k} + ... + r^s_{t+1} = a_0 + a_1\,\xi_t + u_{t+k,t}$, where k denotes the length of forecasting horizon and $u_{t+k,t}$ denotes the overlapping residual. The sample sizes involving predictors ξ^{pat} and ξ^{rd} are 1977Q1–2004Q3 and 1991Q2–2004Q3, respectively. Numbers in brackets are t-statistics based on Hodrick 1B (1992) standard errors that are designed for cumulative predictive regression. Numbers in boldface indicate significance under 5% level (one-sided) with 1B standard errors.

	Panel A: Future 4-Quarter Returns			
♯	const.	stand ξ^{pat}	stand ξ^{rd}	$adj - R^2$
1	0.03	**0.07**		0.18
	[0.81]	[2.42]		
2	0.03		**0.06**	0.05
	[0.99]		[2.43]	
	Panel B: Future 8-Quarter Returns			
♯	const.	stand ξ^{pat}	stand ξ^{rd}	$adj - R^2$
3	0.05	**0.07**		0.08
	[0.75]	[1.67]		
4	0.06		**0.07**	0.02
	[0.93]		[2.03]	
	Panel C: Future 12-Quarter Returns			
♯	const.	ξ^{pat}	ξ^{rd}	$adj - R^2$
5	0.07	**0.11**		0.12
	[0.72]	[2.14]		
6	0.06		**0.13**	0.03
	[0.61]		[3.26]	

Table 7: Forecasting for risk-free asset returns with patent shocks and R&D shocks

In Panel A, I run the short-term predictive regression by regressing the inflation-adjusted risk-free asset returns of time $t+1$, r^f_{t+1}, on the constant term and lagged standardized patent shocks ξ^{pat}_t or lagged standardized R&D shocks ξ^{rd}_t. Numbers in parentheses are the t-statistics of Newey-West's estimator (1987), adjusted for serial correlation and heteroskedasticity up to three lags. Numbers in bold indicate p-values of t-statistics (one-sided) that are smaller than 5%.

Panel A: Patent shocks, 1984Q1–2004Q3			
♯	const.	stand. ξ^{pat}_t	adj-R^2
1	**0.0063**	**0.0013**	0.07
	(6.72)	(2.09)	
Panel B: Patent shocks, 1990Q1–2004Q3			
♯	const.	stand. ξ^{pat}_t	adj-R^2
2	**0.0049**	**0.0011**	0.07
	(8.37)	(2.37)	
Panel C: R&D shocks, 1991Q2–2004Q3			
♯	const.	stand. ξ^{rd}_t	adj-R^2
3	**0.0041**	**0.0033**	0.24
	(6.93)	(4.20)	

Table 8: Short-term forecasting for CRSP index returns: 1977Q1–1995Q4

This table analyzes the predictive regressions in the sample period 1977Q1–1995Q4. I regress the CRSP inflation-adjusted returns and excess returns of time $t+1$ on patent shocks and other predictors in time t (1-step ahead forecasting). Panel A is for simple return case and Panel B is for excess return case. "Lag Ret" denotes the lagged returns. The descriptions of all other predictive variables can be found in the context or in Appendix B. Numbers in parentheses are the t-statistics of Newey-West's estimator (1987), adjusted for serial correlation and heteroskedasticity up to four lags. I use the standardized patent shocks as the predictor in Panel A, and the original patent shocks as the predictor in Panel B. Numbers in bold indicate p-values of t-statistics (one-sided) that are smaller than 5%.

				Panel A: Simple returns					
♯	Const.	Lag Ret	Stand. ξ^{pat}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
1	**0.03**		**0.02**						0.04
	(3.36)		(2.92)						
2	**0.03**	-0.02	**0.02**						0.02
	(3.77)	(-0.19)	(2.86)						
3	**-1.22**		**0.02**	**2.05**					0.06
	(-1.89)		(2.49)	(1.94)					
4	-0.10		**0.03**		0.14				0.03
	(-0.47)		(2.60)		(0.59)				
5	**0.03**		**0.02**			**-3.80**			0.06
	(3.80)		(3.21)			(-1.95)			
6	**0.28**		**0.04**				**0.08**		0.08
	(2.45)		(3.59)				(2.23)		
7	0.04		**0.02**					0.02	0.03
	(1.44)		(2.86)					(0.47)	
				Panel B: Excess returns					
♯	Const.	Lag Ret	Stand. ξ^{pat}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
8	**0.02**		**0.02**						0.03
	(2.38)		(2.69)						
9	**0.02**	-0.01	**0.02**						0.02
	(2.54)	(-0.17)	(2.70)						
10	**-1.27**		**0.02**	**2.11**					0.06
	(-2.01)		(2.32)	(2.04)					
11	-0.12		**0.03**		0.15				0.02
	(-0.56)		(2.46)		(0.65)				
12	**0.02**		**0.02**			**-3.81**			0.06
	(2.68)		(2.96)			(-1.95)			
13	**0.26**		**0.04**				**0.07**		0.07
	(2.16)		(3.30)				(2.00)		
14	0.03		**0.02**					0.02	0.02
	(1.27)		(2.66)					(0.59)	

Table 9: Rolling regression forecasting for CRSP index and excess returns

To inspect the time-variant magnitude of predictability, I regress the logarithmic CRSP inflation-adjusted simple returns and excess returns of time $t+1$ on standardized patent shocks in time t with a rolling 80-quarter window: $r^s_{t+1} = a^s + b^s \xi^{pat}_t + e^s_{t+1}$; $r^e_{t+1} = a^e + b^e \xi^{pat}_t + e^e_{t+1}$. Panel A reports the results of simple returns, and Panel B reports the results of excess returns. t-statistics is based onNewey-West's estimator (1987), adjusted for serial correlation and heteroskedasticity up to three lags. p-values of t-statistics are for one-sided testing.

Panel A: Simple returns				
Sample Period	$\hat{b^s}$	$t(\hat{b^s})$	p-values	$adjR^2$
1976Q1 - 1995Q4	0.024	2.92	0.002	0.037
1977Q1 - 1996Q4	0.021	2.51	0.007	0.028
1978Q1 - 1997Q4	0.020	3.20	0.001	0.043
1979Q1 - 1998Q4	0.018	2.38	0.010	0.028
1980Q1 - 1999Q4	0.018	2.32	0.012	0.027
1981Q1 - 2000Q4	0.014	1.78	0.040	0.014
1982Q1 - 2001Q4	0.014	1.92	0.029	0.012
1983Q1 - 2002Q4	0.022	2.64	0.005	0.062
1984Q1 - 2003Q4	0.022	2.74	0.004	0.066
1985Q1 - 2004Q4	0.022	2.72	0.004	0.065
Panel B: Excess returns				
Sample Period	$\hat{b^e}$	$t(\hat{b^e})$	p-values	$adjR^2$
1976Q1 - 1995Q4	0.027	3.02	0.002	0.050
1977Q1 - 1996Q4	0.023	2.57	0.006	0.037
1978Q1 - 1997Q4	0.023	3.25	0.001	0.055
1979Q1 - 1998Q4	0.020	2.45	0.008	0.037
1980Q1 - 1999Q4	0.019	2.26	0.013	0.033
1981Q1 - 2000Q4	0.016	1.78	0.039	0.018
1982Q1 - 2001Q4	0.014	1.85	0.034	0.013
1983Q1 - 2002Q4	0.021	2.60	0.006	0.054
1984Q1 - 2003Q4	0.022	2.74	0.004	0.061
1985Q1 - 2004Q4	0.021	2.65	0.005	0.057

Table 10: Two-step-ahead forecasting for CRSP index returns: patent shocks and others

To accommodate the reporting lag, I examine the predictability of patent shocks and other predictors for 2-step ahead stock returns in sample period 1977Q2–2004Q3. Here I regress the logarithmic CRSP inflation-adjusted index returns of time $t+2$ on patent shocks and other predictors in time t: $r^s_{t+2} = X_t\beta + e_{t+2}$. "Lag Ret" denotes the r^s_t. I use the standardized patent shocks as predictor in Panel A, and the original patent shocks as predictor in Panel B. Numbers in parentheses are the t-statistics of Newey-West's estimator (1987), adjusted for serial correlation and heteroskedasticity up to four lags. Numbers in bold indicate p-values of t-statistics (one-sided) that are smaller than 5%.

Panel A: Standardized patent shocks and other predictors

♯	Const.	Lag Ret	Stand. ξ^{pat}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
1	**0.02**		**0.02**						0.03
	(3.10)		(2.65)						
2	**0.02**	-0.04	**0.02**						0.03
	(2.86)	(-0.48)	(2.63)						
3	-0.42		**0.02**	0.74					0.04
	(-1.00)		(2.47)	(1.06)					
4	-0.14		**0.02**		0.18				0.03
	(-0.79)		(2.64)		(0.93)				
5	**0.02**		**0.02**			-1.82			0.03
	(3.11)		(2.78)			(-0.76)			
6	**0.11**		**0.02**				0.02		0.04
	(1.73)		(2.56)				(1.34)		
7	0.03		**0.02**					0.01	0.03
	(0.97)		(2.74)					(0.28)	

Panel B: Patent shocks and other predictors

♯	Const.	Lag Ret	ξ^{pat}	cay	SW	RRel	$d-p$	$d-e$	$adjR^2$
8	**0.02**		**60.58**						0.03
	(2.45)		(2.65)						
9	**0.02**	-0.04	**63.45**						0.03
	(2.32)	(-0.48)	(2.63)						
10	-0.43		**58.81**	0.74					0.04
	(-1.01)		(2.47)	(1.06)					
11	-0.14		**64.14**		0.18				0.03
	(-0.81)		(2.64)		(0.93)				
12	**0.02**		**63.27**			-1.82			0.03
	(2.42)		(2.78)			(-0.76)			
13	**0.10**		**63.20**				0.02		0.04
	(1.66)		(2.56)				(1.34)		
14	0.03		**62.16**					0.01	0.03
	(0.84)		(2.75)					(0.28)	

Table 11: Check for possible small sample biases

Here I check the correlation between predictive regression residuals and predictor's innovations because Stambaugh (1986, 1999) showed that the predictive regressor's coefficient is upward biased and may deviate substantially from the standard regression setting. In this table, I report possible small sample biases of the predictability of the patent shocks (ξ^{pat}) and R&D shocks (ξ^{rd}). In Panel A, I estimate an AR(1) model for ξ: $\xi_{t+1} = a_0 + a_1 \xi_t + \epsilon^{\xi}_{t+1}$. In Panel B, I use OLS regression to estimate the predictability of CRSP index returns, $r^s_{t+1} = b_0 + b_1 \xi_t + \epsilon^r_{t+1}$. In Panel C, I regress the residuals obtained in Panel B on AR(1) residuals obtained in Panel A, $\epsilon^r_{t+1} = c_0 + c_1 \epsilon^{\xi}_{t+1} + e_{t+1}$. Finally, I assume that the downward bias of a_1 is $-(1+3a_1)/T$, and the biases in b_1 can be estimated as $Bias = -c_1(1+3a_1)/T$ and are reported in Panel B for comparison. The corrected b_1^0 is therefore $b_1 - Bias$. All t-statistics are based on Newey-West's (1987) standard errors. Sample periods: 1977Q1–2004Q3 for patent shocks, and 1991Q2–2004Q3 for R&D shocks.

Panel A	**AR(1) structure of ξ:** $\xi_{t+1} = a_0 + a_1 \xi_t + \epsilon^{\xi}_{t+1}$				
	a_0	a_1	$t(a_1)$	$adjR^2$	
ξ^{pat}	0.012	0.640	8.37	0.41	
ξ^{rd}	-0.023	0.864	8.65	0.59	
Panel B	**Predictive regression for CRSP:** $r^s_{t+1} = b_0 + b_1 \xi_t + \epsilon^r_{t+1}$				
	b_0	b_1	$t(b_1)$	$adjR^2$	$Bias$
ξ^{pat}	0.021	0.020	3.38	0.05	0.00029
ξ^{rd}	0.015	0.028	2.02	0.04	0.00047
Panel C	$\epsilon^r_{t+1} = c_0 + c_1 \epsilon^{\xi}_{t+1} + e_{t+1}$				
	c_0	c_1	$t(c_1)$	$adjR^2$	
ξ^{pat}	0.000	-0.011	-1.03	0.00	
ξ^{rd}	0.000	-0.006	-0.30	-0.02	

Table 12: GMM esimtaion

In this table, I report the results of GMM test for the null hypothesis composed of several moment conditions. I use the standard two-step procedure to estimate the mean and standard deviations of the parameters in my model, and calculate Hansen's J-test statistic (1982). The moment conditions derived from the Section 3.2 are:

$0 = E_t[\Delta F_{t+1} - (\Delta n_{t+1})^{\alpha_1}(\Delta k_{t+1})^{\alpha_2}\gamma_{t+1}^{\alpha_3}]$

$0 = E_t[k_{t+1} - \beta\alpha_2\, F_t]$

$0 = E_t[R^s_{t+1} - \beta^{-1}\Delta F_{t+1}]$

$0 = E_t[m_{t+1}R^s_{t+1} - 1]$

$0 = E_t[r^s_{t+1} - const_1 - \alpha_2\alpha_3\xi_t]$

where $\triangle$ denotes gross growth rate, pricing kernel $m_{t+1} = \beta(\Delta n_{t+1})^{-\alpha_1}(\Delta k_{t+1})^{-\alpha_2}(\gamma^{pat}_{t+1})^{-\alpha_3}(\Delta\varepsilon_{t+1})^{-1}$, ΔF denotes output growth, Δn denotes labor growth, Δk denotes capital growth, γ^{pat} denotes the patent growth, ξ^{pat} denotes the patent shocks, $\Delta\varepsilon$ denotes the non-technology shock growth, and $r^s = ln(R^s)$. The null hypotheses for free parameters are: $\beta \geq 1$, α_1, α_2, $\alpha_3 \leq 0$, and $const_1 \leq 0$. I use the Newey-West's (1987) covariance matrix estimate with lag number 4 and 8, while the lag 4 is commonly used in the literature according the rule $floor(T^{1/3}) = 4$. The sample period is 1977Q1–2004Q3.

Panel A: Newey-West (lag 4)					
Parameters	Coeff.	Std. Err.	Null	t-stat.	p-value
β	0.971	0.001	1.00	-39.12	0.00
α_1	0.685	0.213	0.00	3.22	0.00
α_2	0.404	0.005	0.00	88.68	0.00
α_3	0.417	0.094	0.00	4.44	0.00
$const_1$	0.031	0.001	0.00	44.87	0.00
J-statistic:	11.59				
Prob($\mathcal{X}^2(20) > J$-statistic):			0.929		
Panel B: Newey-West (lag 8)					
Parameters	Coeff.	Std. Err.	Null	t-stat.	p-value
β	0.971	0.001	1.00	-49.05	0.00
α_1	0.665	0.147	0.00	4.51	0.00
α_2	0.402	0.004	0.00	90.08	0.00
α_3	0.407	0.063	0.00	6.46	0.00
$const_1$	0.031	0.001	0.00	50.72	0.00
J-statistic:	6.60				
Prob($\mathcal{X}^2(20) > J$-statistic):			0.956		

Table 13: Production function specification of U.K.

I run the ordinary least squares regression with and without technology:

Panel A: $ln(\triangle F(n_t, k_t, A_t, \varepsilon_t)) = \alpha_1 ln(\triangle n_t) + \alpha_2 ln(\triangle k_t) + \alpha_3 ln(\gamma_t) + \epsilon_t$,

Panel B: $ln(\triangle F(n_t, k_t, A_t, \varepsilon_t)) = \alpha_1 ln(\triangle n_t) + \alpha_2 ln(\triangle k_t) + \epsilon_t$,

where $\triangle$ denotes gross growth rate (e.g. $\triangle F(n_t, k_t, A_t, \varepsilon_t) = F(n_t, k_t, A_t, \varepsilon_t)/F(n_{t-1}, k_{t-1}, A_{t-1}, \varepsilon_{t-1})$). The production output $F(n_t, k_t, A_t, \varepsilon_t)$ is real GDP per worker; labor is the average weekly work hours divided by hours of five days; and the investment is the real gross fixed capital formation per worker. The proxy for technological growth is British patent growth. Details of data are provided in Appendix C, and the sample period is 1991Q1–2004Q4. The standard errors are adjusted for serial correlation and heteroskedasticity up to three lags by Newey-West's (1987) estimator.

Panel A:	**With technological growth**			
	Coef	Std. error	t-stat.	adj-R^2
Labor (α_1)	1.110	0.160	6.91	0.189
Investment (α_2)	0.031	0.033	0.93	
Technology (α_3)	1.571	0.323	4.86	
Panel B:	**Without technological growth**			
	Coef	Std. error	p-value	adj-R^2
Labor (α_1)	1.288	0.184	7.00	0.081
Investment (α_2)	0.134	0.052	2.57	

Table 14: Forecasting FTSE index returns with British patent shocks

In Panel A, I run the short-term predictive regression by regressing the FTSE100 index returns in logs of time t (r_t^{UK}) on the lagged index returns in logs (r_{t-1}^{UK}) and lagged standardized British patent shocks (ξ_{t-1}^{UKpat}): $r_t^{UK} = a_0 + a_1 r_{t-1}^{UK} + a_2 \xi_{t-1}^{UKpat}$. In Panel B, I run the long-term predictive regression by regressing the cumulative h-period future logarithmic FTSE100 index returns since time t, $r_t^{UK} + r_{t+1}^{UK} + ... + r_{t+h-1}^{UK}$, on lagged British patent shocks (ξ_{t-1}^{UKpat}): $r_t^{UK} + r_{t+1}^{UK} + ... + r_{t+h-1}^{UK} = a_0 + a_2 \xi_{t-1}^{UKpat}$. The sample period is 1991Q1–2004Q4. Numbers in brackets are the t-statistics based on Hodrick 1b (1992) estimation. Numbers in bold indicate p-values of t-statistics (one-sided) that are smaller than 5%.

Panel A: Short-term forecasting				
♯	a_0	a_1	a_2	adj-R^2
1	0.015		**0.021**	0.07
	(1.56)		(2.35)	
2	0.013	-0.122		0.00
	(1.14)	(-1.20)		
3	0.017	-0.167	**0.023**	0.08
	(1.50)	(-1.67)	(2.38)	
Panel B: Long-term forecasting				
♯	a_0	a_1	a_2	adj-R^2
h=4	0.051		0.012	-0.01
	[1.29]		[0.39]	
h=8	0.090		-0.009	-0.01
	[1.13]		[-0.23]	
h=12	0.140		-0.037	0.00
	[1.15]		[-0.74]	

Table 15: Time Series Properties of Calibrated and Historical Data

In the upper panel, I calibrate the economic dynamics of Model 2 for ten simulations, each is of length 600. I report the averages of means and standard deviations of variables from ten simulations. Parameters and initial conditions: $\beta = 0.98$, $\bar{n} = 1$, $n_0 = 0.3$, $\eta = 0.33$, $\delta = -0.1$, $k_0 = 0.5$, $\alpha_0 = 15$, $\alpha_1 = 0.64$, $\alpha_2 = 0.36$, and $\alpha_3 = 0.64$. Norm of the capital partition = 0.01 (i.e. 201 grid points on the range $[0.01, 2.01]$). For technology growth, γ, three possible values are $[1, 1.005, 1.01]$ with probability $p_{i,i} = 0.5$ and $p_{i,j} = 0.25$ where $i \neq j$. Technology shock is $\xi_t = \gamma_t - \gamma_{t-1}$. Two possible values of non-technology shocks are $\overline{\varepsilon} = 1.005$ and $\underline{\varepsilon} = 0.995$, each with 0.5 probability. The labor productivity is defined as $F(n_t, k_t, A_t, \varepsilon_t)/n_t$. The sample period for historical data is 1977Q1–2004Q3 and the data frequency is quarterly; the technological growth is patent growth; the output is real GDP per capita; the capital is real capital per capita; the working time is the weekly working hours divided by total hours in five days; the consumption is real personal consumption expenditures (billions USD) divided by the population (thousands); the stock return is the CRSP value-weighted index return (including dividends); the risk-free asset return is the one-month T-bill return; both risk-free asset return and stock return are inflation-adjusted with the price level. All other variables are the same as described in Appendix B. I also show the average correlation between technology shocks and output growth, next period's stock returns, and next period's excess returns.

	Calibrated Data		**Historical Data**	
Variables	Mean	Standard deviation	Mean	Standard deviation
Technological growth, γ_t	0.005	0.004	0.005	0.001
Output growth, $ln(F_t/F_{t-1})$	0.005	0.008	0.005	0.008
Capital growth, $ln(k_t/k_{t-1})$	0.005	0.004	0.007	0.003
Working time, n_t	0.243	0.000	0.289	0.005
Consumption growth, $ln(c_t/c_{t-1})$	0.005	0.009	0.006	0.006
Productivity growth, $ln(F_t/n_t) - ln(F_{t-1}/n_{t-1})$	0.005	0.008	0.006	0.007
Risk-free asset return, $R_t^f - 1$	0.013	0.004	0.006	0.006
Stock return, $R_t^s - 1$	0.015	0.004	0.022	0.083
$Corr(ln(F_t/F_{t-1}), \xi_t)$	0.883		0.029	
$Corr(R_{t+1}^s, \xi_t)$	0.050		0.231	
$Corr(R_{t+1}^s - R_{t+1}^f, \xi_t)$	0.006		0.239	

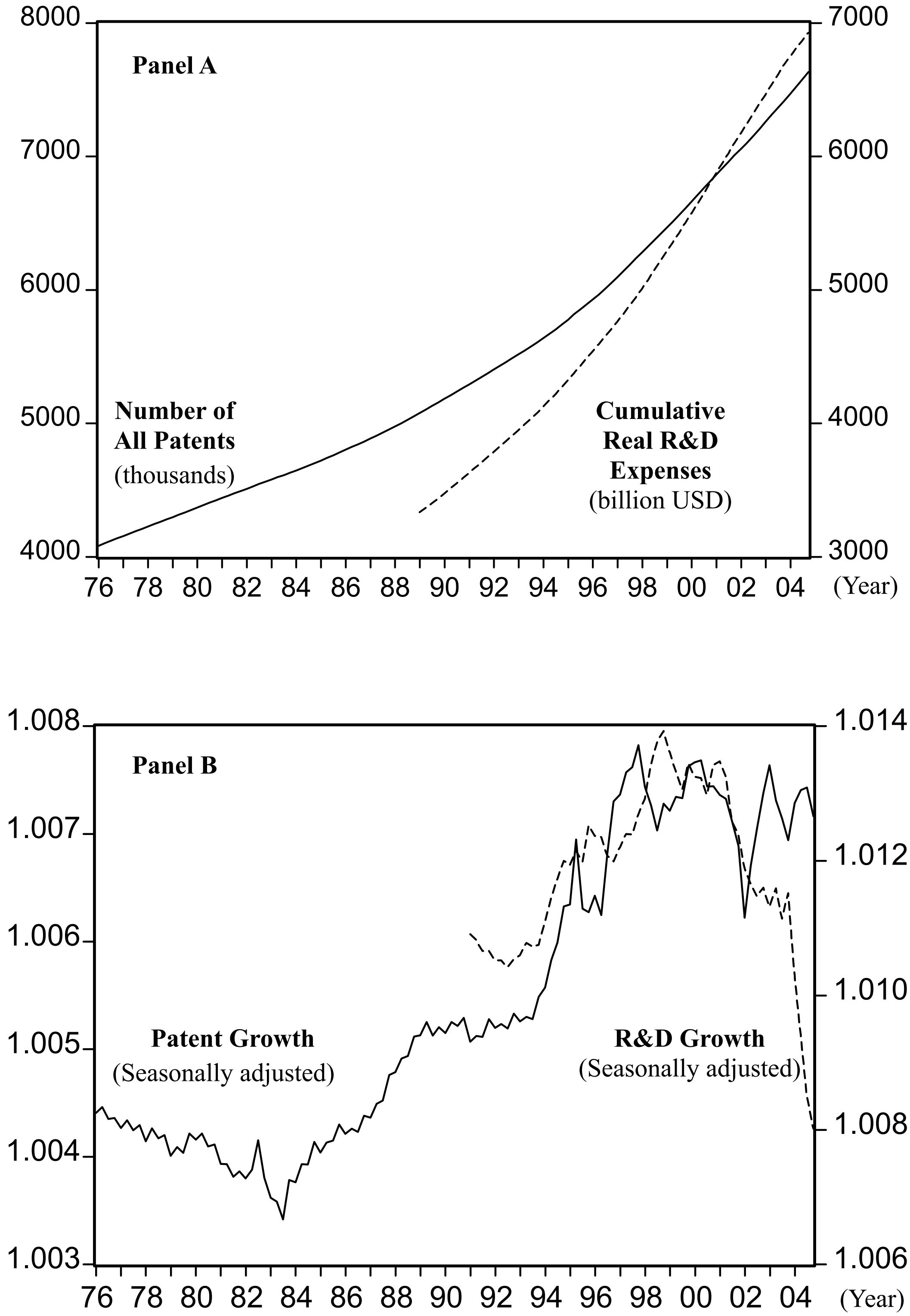


Figure 1. Accumulation and growth of U.S. patents and R&D expenses

Panel A: The solid line denotes the number of total successful patent applications (in thousands), and the dashed line denotes the cumulative real R&D expenses (in billions of USD). Panel B: The solid line denotes the growth of total successful patent applications, and the dotted line denotes the growth of cumulative real industrial R&D expenses.

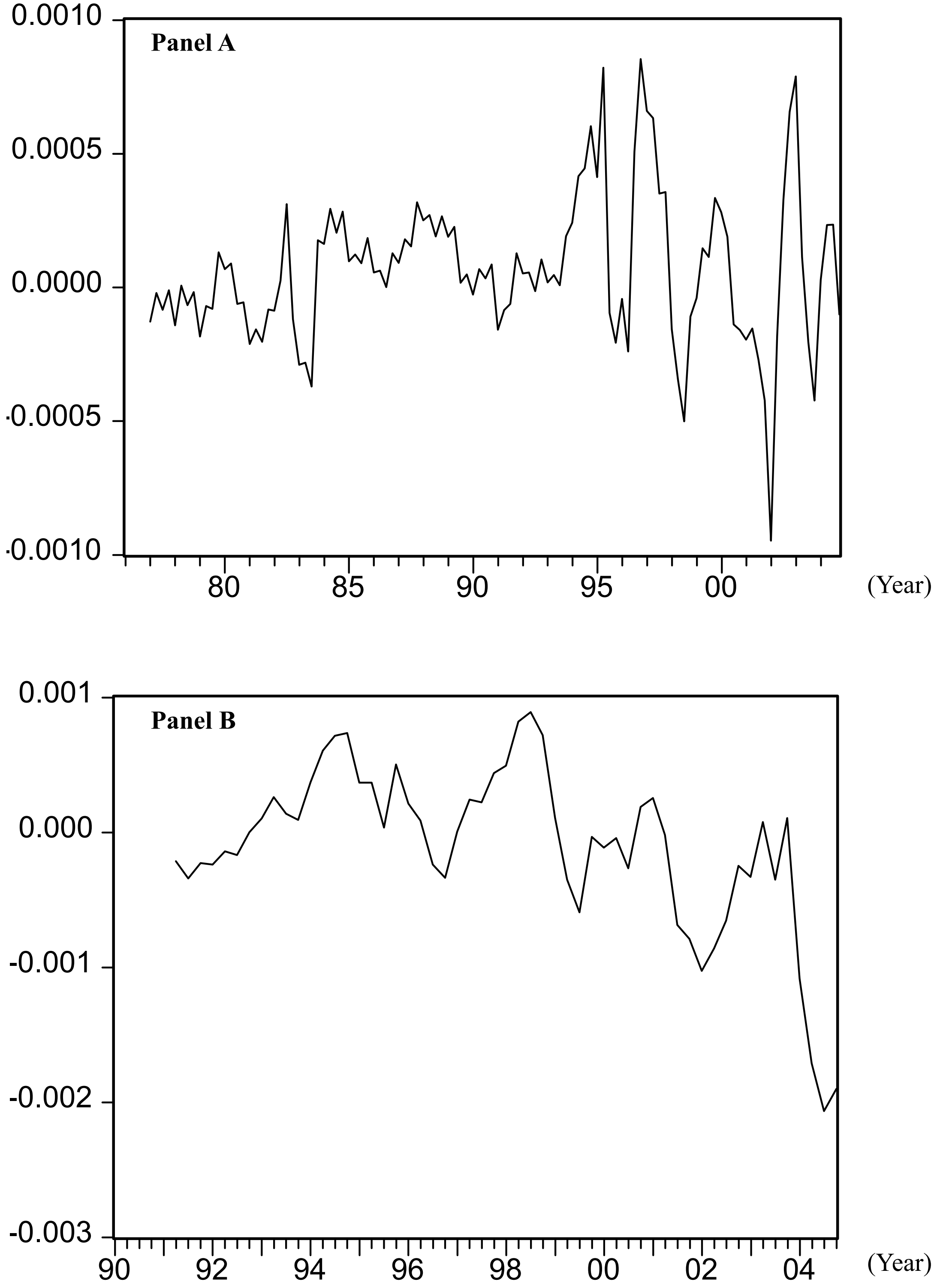
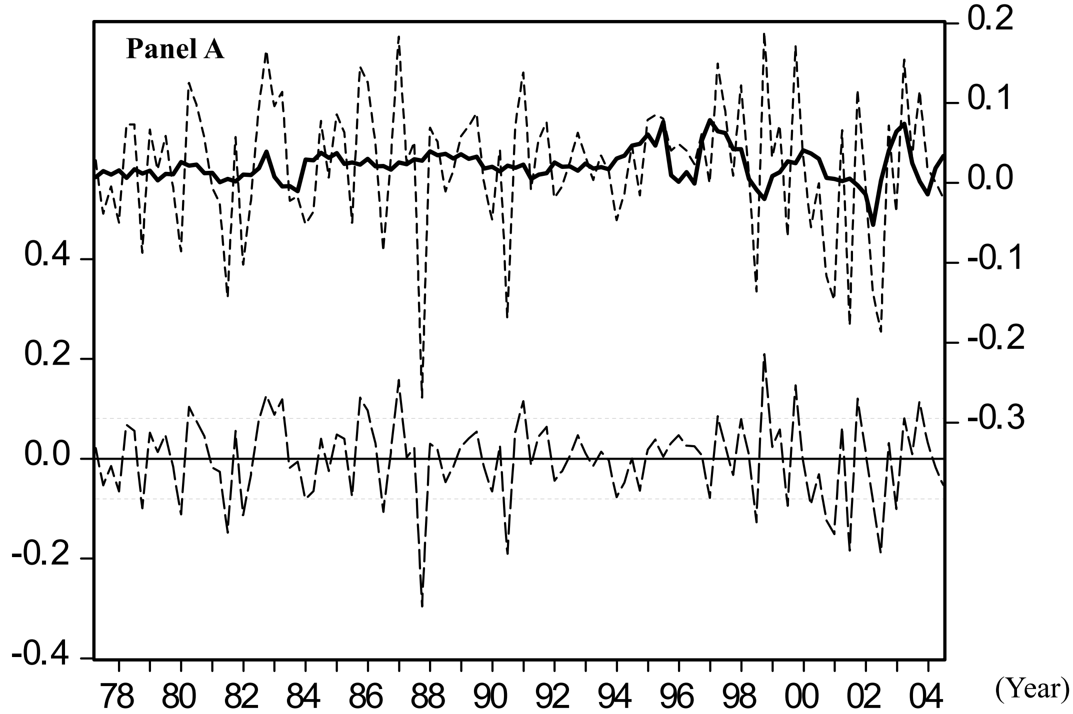


Figure 2. Time series of U.S. patent shocks and R&D shocks

Panel A: U.S. patent shocks; Panel B: U.S. R&D shocks. The details are provided in the context.



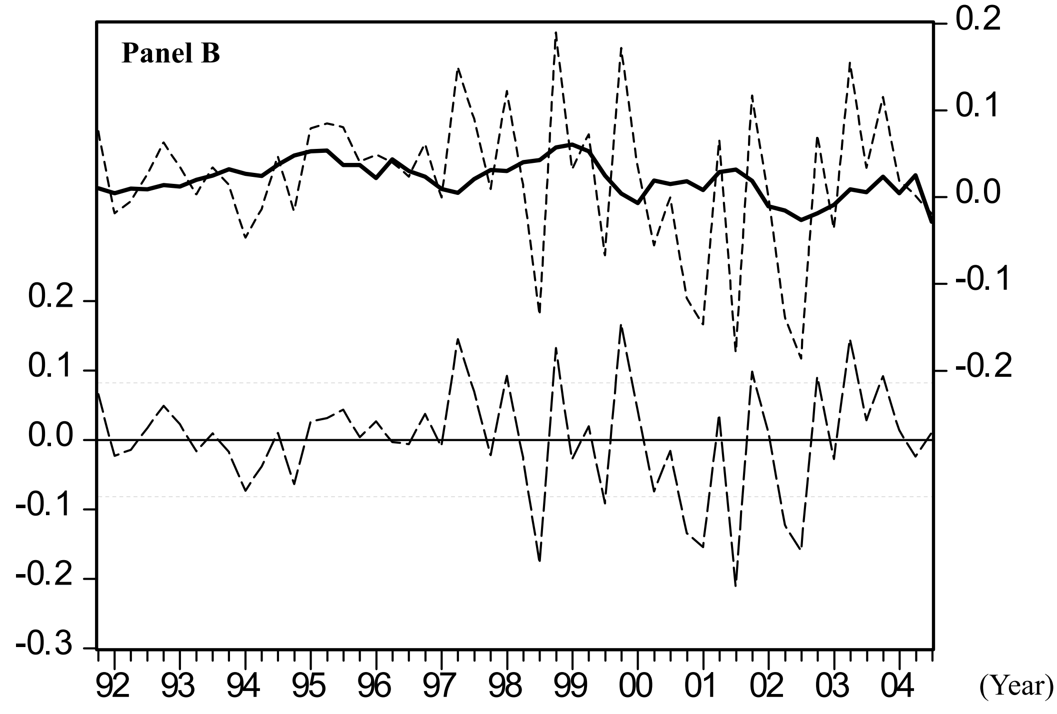


Figure 3. CRSP index returns and forecasts based on technology shocks

The predictor of Panel A is the standardized patent shocks, and the predictor of the Panel B is the standardized R&D shocks. The bold solid line and dotted line on the top of each figure denote the forecasted and realized log CRSP index returns, respectively. The forecasts are computed by 1-step ahead predictive regression with the technology shocks and an intercept (regression #1 in Table 2 and 3). The dashed line on the bottom of each figure denotes the predictive residuals. Left vertical axis is for predictive residuals, and right vertical axis is for realized returns and forecasts.

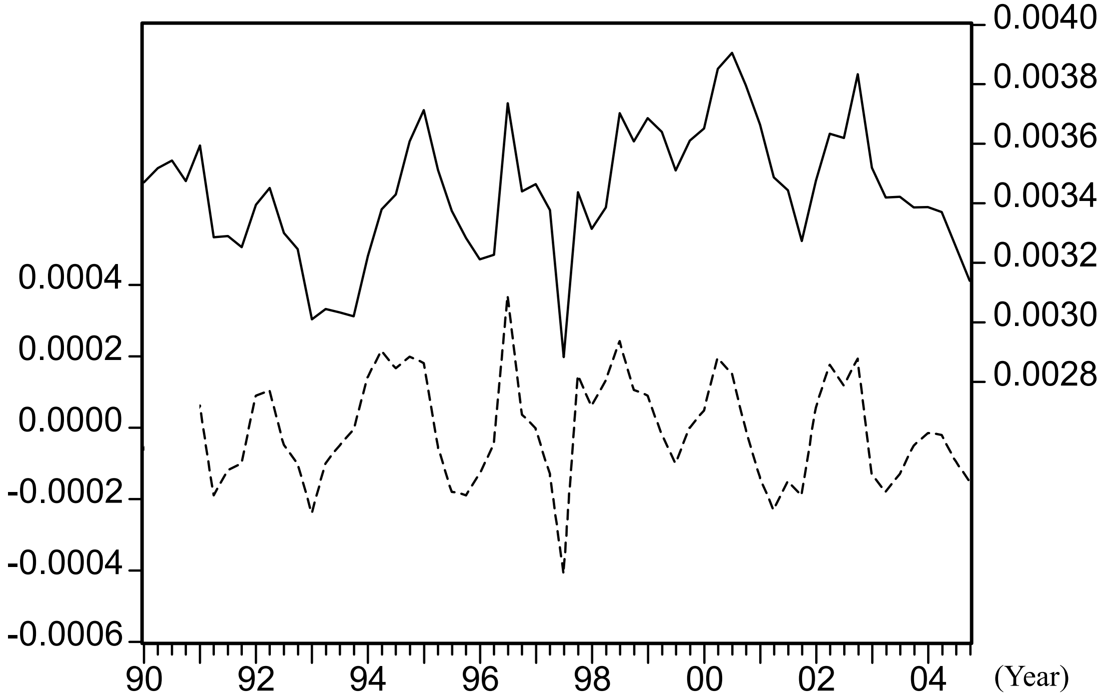


Figure 4. Time series of U.K. patent growth and shocks

The solid line denotes the time series of seasonally adjusted growth of total British patent applications, and the dashed line denotes the times series of British patent shocks.

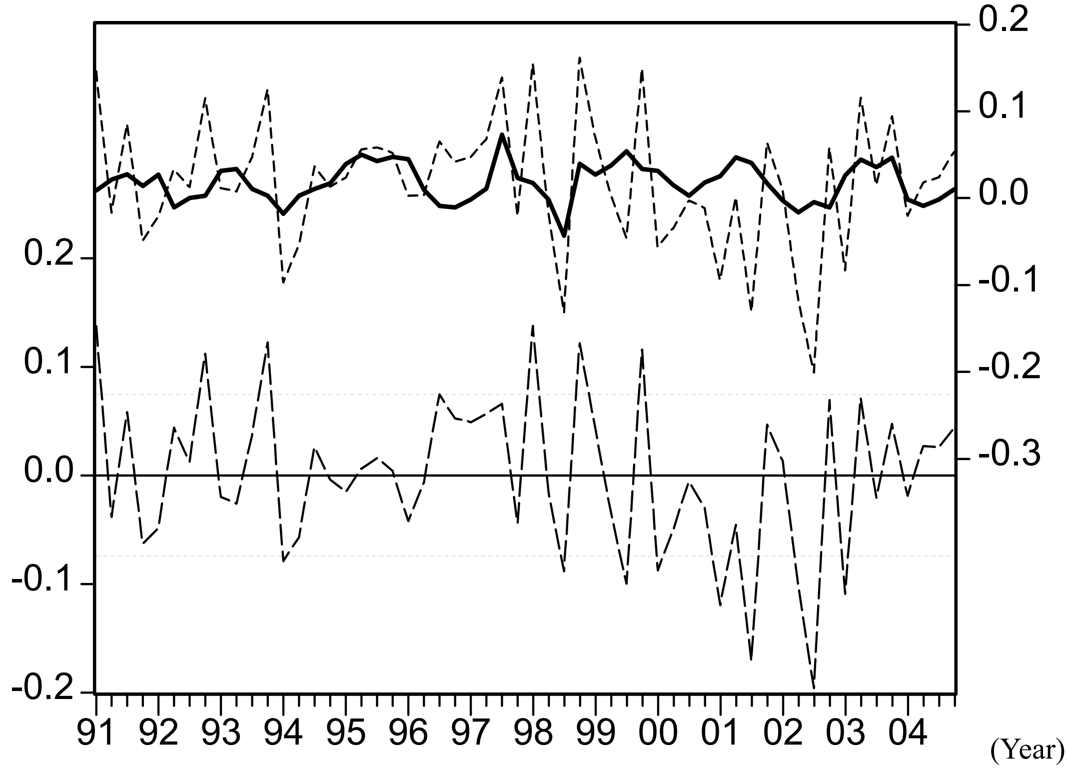


Figure 5. FTSE100 index returns and forecasts based on U.K. technology shocks

The bold solid line and dotted line on the top of each figure denote the forecasted and realized log FTSE100 index returns, respectively. The forecasts are obtained from 1-step ahead predictive regression using U.K. technology shocks and a constant (regression #1 of Table 12). The dashed line on the bottom of figure denotes the predictive residuals. Left vertical axis is for predictive residuals, and right vertical axis is for realized returns and forecasts.