Online Appendix of

Patent Thickets, Stock Returns,

and Conditional CAPM

A Institutional Background of Patent Thickets

A legal definition of patent thickets will be overlapping patent rights. Patent rights are defined and bounded by "claims" made in granted patent documents. Claims specify the exact subject matter for which protection is sought and give rise to exclusive rights (Christie and Dent, 2010).[1] When patent examiners in the U.S. Patent and Trademark Office (USPTO) examine patent applications, they evaluate if patent claims meet the requirements for patent validity. Claims of different patents may partially overlap if a new patent's claim is a significant refinement (with novelty) of that of an old patent, which results in overlapping patent rights. For example, even if Patentee A has a patent claiming a mixture that contains substances E and F, Patentee B can still patent a claim of the mixture wherein the proportion of F is in the range of $30 - 50\%$ for particular purposes. Nevertheless, when Patentee B plans to commercialize its patent for profits in ways covered by such overlapping claims, it negotiates with Patentee A for licensing, or potentially faces patent litigations.

Figure OA.I in Online Appendix illustrates the concept of patent thickets in which each box denotes the claim coverage of a patent. Given two patents (Patents X and Y) that are ex ante identical in future payoffs and that both overlap with four other patents, the firm owning Patent X overlaps with *two* firms (I and J) in patent rights, and the firm owning Patent Y overlaps with *four* firms (K, L, M, and N) in patent rights. The latter firm is thus subject to a deeper patent thicket than the former one, as the commercialization of (the overlapping claims of) Patent Y is more costly because its owner must negotiate with two more firms for licensing fees, or else potentially faces litigations from them. According

[1] For example, the first claim (out of a total of 13) of an important optical touchscreen patent "Infrared light beam x-y position encoder for display devices" (US 3775560) is "An x-y coordinate position address encoder for display devices comprising: an array of a plurality of infrared sources and detectors mounted in a paired manner along respective sides of said display device to provide respective crossing beams in the x and y coordinate directions adjacent the surface of said display device; means, coupled to said plurality of infrared sources and detectors, for sequentially activating pairs of said sources and detectors for beam scanning the surface of said display device in the x direction while simultaneously sequentially activating pairs of said sources and detectors for beam scanning the surface of said display device in the y direction; and address means for responding to an interruption of said crossing beams and providing the x and y address of the position of said interruption." (https://www.google.com.hk/patents/US3775560?dq=3775560)

to Trajtenberg (1990, page 174), "*Moreover, there is a legal dimension to patent citations, since they represent a limitation on the scope of the property rights established by a patent's claims, which carry weight in court. Equally important, the process of arriving at the final list of references, which involves the applicant and his attorney as well as the examiner, apparently does generate the right incentives to have all relevant patents cited, and only those.*" When a patent application contains claims close to those of a prior patent, it must cite the prior patent in order to justify its novelty. For example, the patent application for a mixture containing substances E and F wherein the proportion of F is in the range of 30-50% must cite the prior patent claiming a mixture containing substances E and F (Christie and Dent, 2010). Thus, the reference list provided in a patent document allows us to track the owners of prior patents with overlapping claims with whom a firm may have to negotiate in order to commercialize its patents. The connection between references and overlapping patent rights (and subsequent patent litigations) is further supported by Lanjouw and Schankerman (2001) and Watzinger et al. (2017). The former finds that firms tend to initiate infringement litigations against other firms which cite their patents, and the latter shows that the enforced free-licensing of Bell's patents in 1956 leads to a significant increase of new patents that cite Bell's prior patents.

B Robustness Checks

B.1 Controlling for Other Return Predictors

Furthermore, we examine whether the return predictability associated with patent thickets is distinct from other well-known return predictors, including size, book-to-market ratio, momentum, and reversal. We construct two-way sorted, five-by-three industry-balanced portfolios on our PT measure and one of these predictors. We then calculate the value-weighted excess returns of these 15 portfolios from July of year t to June of year $t+1$.

The average returns of these 15 portfolios are reported in Table OA.IV in Online Appendix. We first focus on Panel A, which controls for firm size. In the upper part of Panel A, we report the average excess returns of 15 portfolios and five particular PT quintile portfolios that average excess returns of all three size groups (Small, 2, and Big) in the same PT quintile (in the column labelled "Average"). The returns presented in the average can be regarded as "characteristic-adjusted" portfolio returns that have controlled for the control variable (see Fama and French, 1993). We find that the top-quintile PT portfolio underperforms the bottom-quintile PT portfolio in average returns in the small, big, and average columns.

In addition, we construct the following four long-short portfolios to examine if the PT effect remains after we control for the size effect. Within each size group (Small, 2, and Big), we form a high-minus-low (High–Low) portfolio by taking a long position in the top-quintile PT portfolio and a short position in the bottom-quintile PT portfolio. The returns of these three portfolios are used to measure the return predictability associated with PT within each size group. Moreover, we construct the fourth portfolio (the average High–Low portfolio) by taking a one-third position in the top-quintile PT portfolio of each size group and shorting a one-third position in the bottom-quintile PT portfolio of each size group. The monthly return of this portfolio is equivalent to the average of three High–Low portfolios' monthly returns. More importantly, the average of this portfolio's monthly returns can be regarded as the PT predictability net of the influence of the size effect (e.g., Fama and French, 1993).

In the lower part of Panel A, the first two rows present the average returns and standard errors of the High–Low portfolios across four size groups (Small, 2, Big, and Average). Although these returns are not always significantly negative, the average return on the average High–Low portfolio is -0.41% with statistical significance at the 1% level. We further control for systematic risk by regressing the returns of each High–Low portfolio on common systematic risk factors, and report the alphas of these factor models. We find that all alphas are negative; more importantly, the alphas of the average High–Low portfolio are significantly negative in all factor models. These results collectively suggest that the PT predictability is not a small-size effect, as the High–Low portfolio provides significantly negative returns and alphas among big firms.

In Panel B of Table OA.IV in Online Appendix, we control for the value effect (B/M) following the same procedure that we use to construct Panel A. We find that the PT predictability remains when we control for the book-to-market ratio. The average returns and alphas of all High–Low portfolios are negative. One noteworthy observation is that the PT predictability seems to focus on growth firms, which is perhaps not surprising given that patents stand for growth options. Panels C and D of Table OA.IV in Online Appendix show that the PT predictability prevails in high momentum and high reversal groups. Overall, the returns on the High–Low portfolio remains significant in all average columns in this table, suggesting that the PT predictability exists after we control for other common return predictors.

B.2 Controlling for Other Innovation Variables and Product Competition

We also examine whether the PT predictability is distinct from characteristics such as a focal firm's patent portfolio size (CTBE), its counterparties' patent portfolio size (CPPS), the quality of its patent portfolio (PQ), and R&D intensity (RDBE) in predicting stock returns. Panels A to D of Table OA.V in Online Appendix suggest that the returns on the

High–Low portfolio remain significant in all average columns, even when we control for these innovation-related variables. In sum, these analyses suggest that neither common return predictors nor innovation-related firm characteristics can eliminate the return predictive power of patent thickets.

It is also important to examine the robust role of patent thickets when we control for product competition. To do so, we conduct two-way portfolio sorting on patent thickets and one of the three measures of product market competition: sales competition (SC), product similarity (PS), and product fluidity (PF). We present our results in Table OA.VI in Online Appendix, which shows that the return spread of patent thickets does not concentrate in high or low groups only, and remains robust when we control for the three different measures of product market competition.

B.3 Controlling for Technological Complementarity

We argue that deeper patent thickets predict lower stock returns as they reflect coordination difficulty among previous patent owners. However, the PT predictability may be weakened when the previous patent owners are able to coordinate more effectively. We hypothesize that technological complementarity among them may provide certain incentives for coordination and thereby mitigate the rent-extracting problem of patent thickets. We thus use the mutual citations among previous patent owners to measure technological complementarity because firms mutually citing each other in their patents tend to cooperate more (e.g., alliances, mergers) and achieve better collaborative outcomes, following Mowery et al. (1998) and Makri et al. (2010).

In Table OA.VII in Online Appendix, we conduct two-way portfolio sorting on patent thickets and one of the two measures of technology complementarity (MC1 and MC2). As shown in Panels A and B, the return spread of patent thickets remains negative and statistically significant in the subgroup of firms with the lowest and median technological complementarity among previous patent owners. It also becomes statistically insignificant when the previous patent owners' technological complementarity falls into the highest tercile. These results confirm the intuition that, when there is better coordination among previous patent owners, patent thickets are less a concern and become less related with expected stock returns.

B.4 Fama-MacBeth Regressions

We also implement monthly Fama-MacBeth regressions to examine if the negative PT-return relation is incrementally robust to other known predictors. This analysis allows us to control more extensively for other characteristics that can predict returns or reflect firms' exposures to systematic risk. For each month of from July of year $t+1$ to June of year $t+2$, we regress the monthly stock returns in excess of one-month Treasury bill rates on the quintile ranks of PT at the end of year t. The quintile rank of PT ranges from one to five is based on within-industry ranks in Panel A (similar to Table VI) or among all firms in Panel B (similar to Table OA.III in Online Appendix). We use PT ranks to be consistent with our portfolio analysis. We also control for market size in logarithm, book-to-market ratio in logarithm, momentum (MOM11), reversal (REV), patent portfolio size (CTBE), counterparties' patent portfolio size (CPPS), patent quality (PQ), R&D intensity (RDBE), technology complementarity (MC1), and product competition (SC). All variables have been defined in Section 2.2. Table OA.VIII in Online Appendix shows that PT is able to predict stock returns We find that PT ranks significantly and negatively predict stock returns. In Panels A and B, the coefficients on PT ranks are 0.03 and 0.04 in percentage, respectively, suggesting that when a firm's PT increases by one quintile rank, its monthly stock return increases by $0.03\% - 0.04\%$.

B.5 Robustness Checks for Alternative Measure of Patent Thickets

Our current measure of patent thickets based on the number of patents is possibly subject to biases caused by strategic patenting (i.e., firms may file many patents or cut an important patent into several incremental patents; see Cockburn and MacGarvie, 2011; Noel and Schankerman, 2013). To alleviate such concerns, we construct an alternative measure of patent thickets based on the number of forward citations of cited patents.[1] Specifically, we redefine $Numcites_{i,t}^{j}$ in Equation (1), based on the number of total forward citations accumulated until the calculation year t that go to the previous patent owner j's patents, which are cited by the focal firm i's five-year patent portfolio by year t.[2] Accordingly, $Numcites_{i,t}$ is replaced by the summation of $Numcites_{i,t}^{j}$ across all previous patent owners. We implement the one-way portfolio analyses with the new citation-based patent thickets measure and present our results in Table OA.IX in Online Appendix. Empirical evidence implies that the return predictive power of patent thickets is robust to strategic patenting behaviors.

We acknowledge that, as a firm observes a deeper patent thicket ex ante (Lemley and Shapiro, 2005), it will only invest in a high-quality project, which will probably result in a high-quality patent. As we discussed earlier, a high-quality patent carries high option value and thus increases a firm's systematic risk exposure and expected stock return (i.e., patenting-behavior channel). However, our empirical evidence from two-way sorted portfolios does not support this argument. Thus, the association between observed patent thickets and firms' choices with respect to innovation cannot explain a negative patent thicket-return relation.

We further consider an *unexpected* patent thicket measure that is based on cited patents

[1] Cockburn and MacGarvie (2011) suggest that forward citations are the real measure of technological value; thus, a patent thicket measure based on forward citations is immune to strategic patenting behaviors (Harhoff et al., 1999).

[2] In the main measure of patent thickets, $Numcites_{i,t}^{j}$ denotes the number of the previous patent owner j's patents that are cited by the focal firm i's five-year patent portfolio by year t.

unknown to the focal firm when it files for patent applications.[3] To construct such a measure, we follow Equation (1) but exclude cited patents that are granted before the application dates of citing patents.[4] This new measure thus captures the degree of patent thickets that are ex ante unknown but ex post revealed to the focal firms. We implement the same test as shown in Table V by replacing our main patent thicket measure with this new one and report our results in Table OA.X in Online Appendix. It is shown that the return spread of this new unexpected patent thicket measure is -0.22% and statistically significant at the 10% level. Admittedly, the magnitude of the return spread becomes lower, probably due to a large reduction in variation of the patent thicket measure.

We also acknowledge that patent thickets change when cited (or citing) patents are transferred to others; however, we do not consider the re-assignment issue in our main analysis because patent re-assignment data from the USPTO is based on voluntary reporting and is thus subject to selection issues and incompleteness. In addition, the recent establishment of patent pools and industry consortia such as Rock Star and RPX facilitates firms' use of patents.[5] We thus construct an alternative patent thicket measure to address the re-assignment issue as a robustness check.[6] When we combine our data with the USPTO patent re-asssignment data, we find that about 15% of the citing and cited patents in our sample are transferred at least once within the five-year period after they are initially granted. We implement a one-way portfolio analysis based on an adjusted PT measure accounting for

[3]Before 2001, the USPTO publishes the information of patent applications only when these patents are approved. After 2001, even though the USPTO publishes the information of patent fillings 18 months after their initial applications, it is still highly uncertain whether and when these patents will eventually be approved (Kogan et al., 2017). In an untabulated analysis, we find that our results remain similar if we use the sample before 2001.

[4]These cited patents are added by the applicants or examiners after the intial filing. 87% of citing-cited patent pairs in our original sample are excluded. To account for missing values in patent thickets when we use this method, we set them to be zero.

[5]See Clarkson (2004) and van Overwalle (2010) for discussions of various patent pools.

[6]For example, Firm A transfers its Patent X to another Firm B in year t, the cash flow rights of Patent X are transferred out, and thereby the patent thicket of Patent X does not matter to Firm A after year t. In another example, if Firm A's Patent Y is not transferred out but its cited Patent Z is transferred from Firm C to Firm D, then Firm A should negotiate with Firm D rather than Firm C when it commercializes Patent Y.

patent transfer issues.[7] Table OA.XI in Online Appendix presents results highly comparable to our main results.

[7] To construct this new patent thicket measure that adjusts for patent transfers of a focal firm at the end of year t, we exclude both 1) the citing patents granted to the focal firm in the previous five years from year $t-4$ to t but transferred to other assignees by year t, and 2) the cited patents (i.e., patents cited by the focal firm's citing patents granted in the previous five years) transferred to other assignees by year t. We simply exclude the cited patents transferred by year t because we cannot accurately identify whether the assignees of the patent transfer data are public firms or not.

C Online Appendix Tables

Table OA.I
Correlation Matrix

This table reports the pooled correlation matrix of all variables. All variables have been defined in Table II.

	PT	CTBE	CPPS	RDBE	PQ	Litigation	PC	ROA	MC1	MC2	SC	PS	PF	Size	B/M	MOM11	REV	D/A	FV
Patent Thicket (PT)	1	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----
Patent Portfolio Size (CTBE)	0.01	1	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----
Counterparties' Patent Portfolio Size (CPPS)	-0.13	-0.02	1	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----
R&D Intensity (RDBE)	-0.01	0.33	0.00	1	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----
Patent Quality (PQ)	-0.15	0.01	-0.01	0.01	1	----	----	----	----	----	----	----	----	----	----	----	----	----	----
Number of Patent Litigation (Litigation)	0.01	0.00	-0.02	0.00	0.05	1	----	----	----	----	----	----	----	----	----	----	----	----	----
Patent Commercialization (PC)	-0.07	-0.01	0.11	0.01	0.04	0.01	1	----	----	----	----	----	----	----	----	----	----	----	----
Profitability (ROA)	-0.07	0.07	0.03	0.06	0.07	-0.03	0.04	1	----	----	----	----	----	----	----	----	----	----	----
Mutual Citations by Patent Owners (MC1)	-0.09	-0.01	0.04	0.00	0.06	-0.01	0.04	0.03	1	----	----	----	----	----	----	----	----	----	----
Mutually Citing Patent Owners (MC2)	-0.12	-0.01	0.17	0.00	0.18	0.00	0.09	0.07	0.32	1	----	----	----	----	----	----	----	----	----
Sales Competition (SC)	-0.01	0.01	0.03	0.02	0.04	0.00	0.05	0.12	0.00	0.03	1	----	----	----	----	----	----	----	----
Product Similarity (PS)	-0.10	0.04	-0.04	0.06	0.01	-0.02	0.03	0.29	-0.04	-0.20	0.22	1	----	----	----	----	----	----	----
Product Fluidity (PF)	-0.04	0.06	-0.02	0.07	0.03	-0.01	0.07	0.32	-0.05	-0.15	0.29	0.65	1	----	----	----	----	----	----
Market Capitalization (Size)	0.02	-0.02	-0.04	-0.01	0.10	0.32	0.00	-0.08	-0.05	-0.14	-0.02	0.03	0.00	1	----	----	----	----	----
Book-to-Market Ratio (B/M)	0.03	-0.04	0.04	-0.03	-0.08	-0.02	-0.03	-0.04	0.02	0.03	-0.07	-0.07	-0.10	-0.08	1	----	----	----	----
11-month Momentum (MOM11)	-0.01	0.03	0.00	0.00	0.01	0.00	-0.03	-0.04	0.00	0.00	0.00	-0.01	0.01	0.00	0.08	1	----	----	----
Reversal (REV)	0.01	0.00	0.00	0.00	0.01	0.00	-0.04	-0.01	-0.01	0.00	0.02	0.02	0.04	-0.01	0.02	-0.03	1	----	----
Leverage Ratio (D/A)	0.04	0.05	-0.03	0.03	-0.06	0.01	-0.06	-0.17	-0.01	-0.07	-0.15	-0.09	-0.14	0.05	0.09	-0.02	-0.03	1	----
Fundamental Volatility (FV)	-0.04	0.04	0.01	0.03	0.04	-0.01	0.07	0.19	0.02	0.07	0.09	0.12	0.18	-0.05	-0.07	0.00	0.00	-0.09	1

Table OA.II

Controlling for IMC and EMI Factors in the One-way Industry-Balanced Portfolio Sorting Analysis

We re-do the portfolio sorting analysis in Table VI and control for the IMC and EMI factors. *IMC* represents the investment-minus-consumption factor by Papanikolaou (2011). *EMI* denotes the innovative efficient-minus-inefficient factor by Hirshleifer et al. (2012). All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively. Sample period: July 1981 to June 2019 for MKT, SMB, HML, RMW, CMA, UMD, *q*-MKT, *q*-ME, *q*-IA, and *q*-ROE, and July 1981 to June 2008 for IMC, and EMI.

PT	FF6+IMC								FF6+EMI							
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	IMC	Alpha	MKT	SMB	HML	RMW	CMA	UMD	EMI
Low	0.44***	0.94***	0.06	-0.22***	-0.14***	0.05	0.01	-0.01	0.38***	0.94***	0.09**	-0.18***	-0.13***	0.07	0.02	0.24***
	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.03)	(0.01)	(0.12)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.03)	(0.06)
2	0.11	0.96***	0.06	-0.13**	0.11**	0.15**	0.05**	-0.02***	0.11	0.96***	0.09**	-0.10*	0.13***	0.19***	0.05**	0.11*
	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.02)	(0.01)	(0.11)	(0.03)	(0.04)	(0.06)	(0.05)	(0.07)	(0.02)	(0.06)
3	0.11	0.97***	-0.05	-0.19***	-0.07*	0.14**	0.00	-0.01	0.06	0.98***	-0.03	-0.19***	-0.04	0.16**	0.00	0.09*
	(0.10)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.01)	(0.10)	(0.03)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.05)
4	0.15	0.97***	-0.16***	-0.29***	-0.11**	0.18***	0.01	0.01	0.07	0.97***	-0.14***	-0.29***	-0.10**	0.21***	0.02	0.25***
	(0.10)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.01)	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.05)
High	0.07	0.95***	-0.14***	-0.20***	-0.12***	0.16***	-0.09***	-0.01**	-0.00	0.96***	-0.10***	-0.18***	-0.08**	0.18***	-0.09***	0.20***
	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.01)	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)	(0.06)	(0.02)	(0.05)
High-Low	-0.37***	0.01	-0.21***	0.03	0.02	0.10	-0.10***	-0.00	-0.38***	0.02	-0.20***	0.01	0.05	0.11	-0.10***	-0.04
	(0.14)	(0.03)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.01)	(0.14)	(0.04)	(0.05)	(0.07)	(0.06)	(0.09)	(0.03)	(0.07)

PT	HXZ+IMC						HXZ+EMI					
	Alpha	*q*-MKT	*q*-ME	*q*-IA	*q*-ROE	IMC	Alpha	*q*-MKT	*q*-ME	*q*-IA	*q*-ROE	EMI
Low	0.46***	0.96***	0.11***	-0.21***	-0.09**	-0.00	0.39***	0.96***	0.14***	-0.18***	-0.09**	0.26***
	(0.12)	(0.03)	(0.04)	(0.06)	(0.04)	(0.01)	(0.12)	(0.03)	(0.04)	(0.06)	(0.04)	(0.06)
2	0.11	0.96***	0.07**	-0.00	0.12***	-0.02***	0.12	0.95***	0.09***	0.05	0.12***	0.11*
	(0.11)	(0.03)	(0.03)	(0.06)	(0.04)	(0.01)	(0.11)	(0.03)	(0.03)	(0.06)	(0.04)	(0.06)
3	0.08	1.00***	-0.01	-0.05	-0.02	-0.00	0.01	1.00***	-0.00	-0.03	0.00	0.11**
	(0.10)	(0.02)	(0.03)	(0.05)	(0.04)	(0.01)	(0.11)	(0.03)	(0.03)	(0.05)	(0.04)	(0.05)
4	0.19*	1.00***	-0.13***	-0.11*	-0.12***	0.01*	0.10	1.00***	-0.12***	-0.10*	-0.11***	0.27***
	(0.11)	(0.03)	(0.03)	(0.06)	(0.04)	(0.01)	(0.11)	(0.03)	(0.03)	(0.06)	(0.04)	(0.06)
High	0.08	0.97***	-0.14***	-0.12**	-0.15***	-0.01**	0.01	0.97***	-0.11***	-0.08*	-0.13***	0.20***
	(0.09)	(0.02)	(0.03)	(0.05)	(0.03)	(0.01)	(0.09)	(0.02)	(0.03)	(0.05)	(0.03)	(0.05)
High-Low	-0.38***	0.00	-0.25***	0.09	-0.06	-0.01	-0.38***	0.01	-0.25***	0.10	-0.04	-0.06
	(0.14)	(0.03)	(0.04)	(0.07)	(0.05)	(0.01)	(0.15)	(0.04)	(0.04)	(0.07)	(0.05)	(0.07)

Table OA.III

One-way Portfolio Sorting on Patent Thickets

We sort firms with non-missing patent thicket (*PT*) into five PT groups (Low, 2, 3, 4, and High) at the end of June of year *t* from 1981 to 2018, based on firms' PT ranks among all firms in year *t* – 1. We also construct a high-minus-low (High–Low) portfolio by holding a long (short) position in the high (low) PT portfolio. We hold all these portfolios over the next twelve months (July of year *t* to June of year *t* + 1) and calculate their value-weighted monthly returns. We also report the monthly returns in excess of one-month Treasury bill rates (*Excess Return*) and their corresponding alphas and betas in different model specifications. Following Fama and French (1993), *MKT* denotes the market factor, *SMB* denotes the size factor, and *HML* represents the value factor. Following Fama and French (2015), we use *RMW* and *CMA* to represent profitability and investment factors, respectively. *UMD* stands for the momentum factor (Carhart, 1997). Following Hou et al. (2014), *q*-MKT, *q*-ME, *q*-IA, and *q*-ROE denote their market factor, size factor, investment factor, and profitability factor, respectively. All returns and alphas are value-weighted and in percentage. The numbers in parentheses denote the standard errors. ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively. Sample period: July 1981 to June 2019.

PT	Excess Return	CAPM		FF3				FF5					
	Time-series Mean	Alpha	MKT	Alpha	MKT	SMB	HML	Alpha	MKT	SMB	HML	RMW	CMA
Low	0.97***	0.30***	1.01***	0.38***	0.95***	0.13***	-0.21***	0.43***	0.94***	0.10***	-0.18***	-0.10**	-0.03
	(0.23)	(0.09)	(0.02)	(0.09)	(0.02)	(0.03)	(0.03)	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)
2	0.88***	0.27***	0.92***	0.29***	0.90***	0.06*	-0.06*	0.13	0.96***	0.11***	-0.21***	0.22***	0.28***
	(0.21)	(0.09)	(0.02)	(0.10)	(0.02)	(0.03)	(0.03)	(0.10)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)
3	0.70***	0.05	0.98***	0.10	0.96***	-0.04	-0.14***	0.11	0.96***	-0.06**	-0.17***	-0.07*	0.09*
	(0.22)	(0.08)	(0.02)	(0.08)	(0.02)	(0.03)	(0.03)	(0.08)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)
4	0.59***	-0.08	1.00***	0.02	0.97***	-0.09***	-0.27***	0.06	0.96***	-0.13***	-0.27***	-0.13***	0.05
	(0.23)	(0.09)	(0.02)	(0.09)	(0.02)	(0.03)	(0.03)	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)
High	0.56***	-0.08	0.96***	-0.06	0.97***	-0.12***	-0.05**	-0.06	0.98***	-0.15***	-0.12***	-0.10***	0.18***
	(0.21)	(0.06)	(0.01)	(0.06)	(0.01)	(0.02)	(0.02)	(0.06)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)
High-Low	-0.41***	-0.39***	-0.04*	-0.45***	0.01	-0.25***	0.15***	-0.49***	0.04	-0.26***	0.07	-0.00	0.20***
	(0.11)	(0.11)	(0.02)	(0.10)	(0.02)	(0.04)	(0.04)	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)

PT	FF6							HXZ				
	Alpha	MKT	SMB	HML	RMW	CMA	UMD	Alpha	*q* -MKT	*q* -ME	*q* -IA	*q* -ROE
Low	0.41***	0.95***	0.10***	-0.16***	-0.11**	-0.05	0.04*	0.45***	0.95***	0.12***	-0.27***	-0.03
	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.09)	(0.02)	(0.03)	(0.05)	(0.04)
2	0.14	0.96***	0.11***	-0.21***	0.22***	0.28***	-0.01	0.21**	0.93***	0.09**	0.01	0.11***
	(0.10)	(0.02)	(0.03)	(0.05)	(0.05)	(0.07)	(0.02)	(0.10)	(0.02)	(0.03)	(0.05)	(0.04)
3	0.13	0.96***	-0.06**	-0.19***	-0.07*	0.10*	-0.03	0.16*	0.96***	-0.05*	-0.11**	-0.05
	(0.08)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.08)	(0.02)	(0.03)	(0.04)	(0.03)
4	0.04	0.97***	-0.14***	-0.24***	-0.14***	0.03	0.04**	0.09	0.97***	-0.12***	-0.25***	-0.04
	(0.09)	(0.02)	(0.03)	(0.04)	(0.04)	(0.06)	(0.02)	(0.10)	(0.02)	(0.03)	(0.05)	(0.04)
High	-0.02	0.96***	-0.15***	-0.16***	-0.08***	0.21***	-0.08***	0.03	0.96***	-0.15***	-0.03	-0.11***
	(0.06)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.01)	(0.07)	(0.02)	(0.02)	(0.04)	(0.03)
High-Low	-0.43***	0.02	-0.24***	-0.00	0.02	0.26***	-0.11***	-0.42***	0.01	-0.27***	0.24***	-0.08*
	(0.11)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.02)	(0.11)	(0.03)	(0.04)	(0.06)	(0.04)

Table OA.IV

Two-way Industry-Balanced Portfolio Sorting on Patent Thickets and Common Return Predictors

We conduct independent double sorts within industries on patent thicket (*PT*) and another firm characteristic into 15 groups based on the quintiles of PT at the end of year $t-1$ and the 30th and 70th percentiles of *SIZE*, *MOM11*, or *REV* in June of year *t*, or *B/M* at the end of year $t-1$. All these variables are defined in Table II. We also construct the "Average" group, which averages the excess returns of the three control variable groups in the same PT group. We then construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each control variable group and hold these portfolios over the next twelve months (July of year *t* to June of year $t+1$) to calculate monthly value-weighted returns. Sample period: July 1981 to June 2019.

Panel A. Conditional Predictive Power on Market Capitalization (Size)

PT \ SIZE	Small	2	Big	Average
Low	3.27	1.61	0.79	1.89
2	3.06	1.50	0.73	1.76
3	2.81	1.40	0.59	1.60
4	2.46	1.52	0.53	1.50
High	2.45	1.48	0.51	1.48
		High-Low		
Excess Return	-0.82***	-0.13	-0.28**	-0.41***
	(0.21)	(0.13)	(0.12)	(0.10)
CAPM Alpha	-0.86***	-0.13	-0.28**	-0.42***
	(0.21)	(0.13)	(0.12)	(0.10)
FF6 Alpha	-0.89***	-0.19	-0.30**	-0.46***
	(0.22)	(0.14)	(0.12)	(0.10)
HXZ Alpha	-0.94***	-0.12	-0.30**	-0.45***
	(0.23)	(0.14)	(0.12)	(0.10)

Panel B. Conditional Predictive Power on Book-to-Market Ratio (B/M)

PT \ B/M	Low	2	High	Average
Low	0.92	0.89	0.89	0.90
2	0.85	0.78	1.08	0.90
3	0.70	0.63	1.23	0.86
4	0.58	0.67	1.00	0.75
High	0.48	0.53	0.54	0.52
		High-Low		
Excess Return	-0.44***	-0.36**	-0.34	-0.38***
	(0.13)	(0.14)	(0.27)	(0.12)
CAPM Alpha	-0.41***	-0.32**	-0.55**	-0.43***
	(0.13)	(0.15)	(0.26)	(0.12)
FF6 Alpha	-0.44***	-0.33**	-0.45	-0.41***
	(0.13)	(0.15)	(0.28)	(0.12)
HXZ Alpha	-0.45***	-0.33**	-0.48*	-0.42***
	(0.13)	(0.15)	(0.28)	(0.12)

Panel C. Conditional Predictive Power on 11-month Momentum (MOM11)

PT \ MOM11	Low	2	High	Average
Low	1.78	0.73	0.93	1.14
2	1.66	0.65	1.00	1.10
3	1.13	0.45	0.73	0.77
4	0.97	0.33	0.83	0.71
High	1.00	0.51	0.53	0.68
		High-Low		
Excess Return	-0.78***	-0.22	-0.40**	-0.46***
	(0.26)	(0.14)	(0.16)	(0.12)
CAPM Alpha	-0.73***	-0.25*	-0.33**	-0.44***
	(0.26)	(0.14)	(0.16)	(0.12)
FF6 Alpha	-0.75***	-0.07	-0.40**	-0.41***
	(0.27)	(0.15)	(0.16)	(0.12)
HXZ Alpha	-0.78***	-0.13	-0.32*	-0.41***
	(0.27)	(0.15)	(0.17)	(0.12)

Panel D. Conditional Predictive Power on Reversal (REV)

PT \ REV	Low	2	High	Average
Low	1.23	0.82	0.92	0.99
2	1.28	0.77	0.80	0.95
3	0.97	0.57	0.75	0.76
4	1.03	0.52	0.53	0.69
High	0.73	0.52	0.46	0.57
		High-Low		
Excess Return	-0.50**	-0.30**	-0.46***	-0.42***
	(0.22)	(0.14)	(0.18)	(0.11)
CAPM Alpha	-0.43**	-0.27*	-0.40**	-0.37***
	(0.22)	(0.14)	(0.18)	(0.11)
FF6 Alpha	-0.52**	-0.30**	-0.54***	-0.45***
	(0.23)	(0.15)	(0.18)	(0.11)
HXZ Alpha	-0.49**	-0.25*	-0.52***	-0.42***
	(0.23)	(0.15)	(0.18)	(0.11)

Table OA.V

Two-way Industry-Balanced Portfolio Sorting on Patent Thickets and Innovation-Related Variables

We conduct independent double sorts within each industry on patent thicket (*PT*) and another firm characteristic into 15 groups based on the quintiles of PT at the end of year $t-1$ and the 30th and 70th percentiles of *CTBE*, *CPPS*, *PQ*, or *RDBE* at the end of year $t-1$. All these variables are defined in Table II. We also construct the "Average" group, which averages the excess returns of the three control variable groups in the same PT group. We then construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each control variable group and hold these portfolios over the next twelve months (July of year t to June of year $t+1$) to calculate monthly value-weighted returns. Sample period: July 1981 to June 2019.

Panel A. Conditional Predictive Power on Patent Portfolio Size (CTBE)

PT \ CTBE	Low	2	High	Average
Low	0.89	0.98	1.85	1.24
2	0.74	0.94	1.52	1.07
3	0.70	0.54	0.98	0.74
4	0.62	0.52	0.92	0.69
High	0.30	0.63	0.75	0.56
		High-Low		
Excess Return	-0.60***	-0.35**	-1.10***	-0.68***
	(0.16)	(0.16)	(0.32)	(0.15)
CAPM Alpha	-0.49***	-0.35**	-1.08***	-0.64***
	(0.16)	(0.17)	(0.32)	(0.15)
FF6 Alpha	-0.70***	-0.29*	-1.17***	-0.72***
	(0.16)	(0.17)	(0.31)	(0.13)
HXZ Alpha	-0.66***	-0.31*	-1.27***	-0.74***
	(0.16)	(0.17)	(0.32)	(0.14)

Panel B. Conditional Predictive Power on Counterparties' Patent Portfolio Size (CPPS)

PT \ CPPS	Low	2	High	Average
Low	0.85	1.44	0.85	1.05
2	0.72	1.10	1.15	0.99
3	0.60	0.90	0.73	0.74
4	0.44	0.97	1.01	0.81
High	0.56	0.58	0.91	0.68
		High-Low		
Excess Return	-0.29**	-0.86***	0.05	-0.37***
	(0.14)	(0.21)	(0.19)	(0.11)
CAPM Alpha	-0.29**	-0.78***	0.01	-0.35***
	(0.14)	(0.21)	(0.19)	(0.11)
FF6 Alpha	-0.31**	-0.84***	0.14	-0.34***
	(0.14)	(0.21)	(0.20)	(0.11)
HXZ Alpha	-0.30**	-0.84***	0.18	-0.32***
	(0.14)	(0.22)	(0.20)	(0.11)

Panel C. Conditional Predictive Power on Patent Quality (PQ)

PT \ PQ	Low	2	High	Average
Low	0.88	1.01	0.92	0.94
2	1.03	0.74	0.94	0.90
3	0.82	0.65	0.71	0.73
4	1.01	0.53	0.53	0.69
High	0.70	0.57	0.47	0.58
		High-Low		
Excess Return	-0.19	-0.44**	-0.46***	-0.36***
	(0.17)	(0.18)	(0.17)	(0.10)
CAPM Alpha	-0.14	-0.39**	-0.47***	-0.33***
	(0.17)	(0.18)	(0.17)	(0.10)
FF6 Alpha	-0.23	-0.51***	-0.39**	-0.38***
	(0.18)	(0.18)	(0.17)	(0.10)
HXZ Alpha	-0.22	-0.57***	-0.34*	-0.37***
	(0.18)	(0.18)	(0.18)	(0.10)

Panel D. Conditional Predictive Power on R&D Intensity (RDBE)

PT \ RDBE	Low	2	High	Average
Low	0.67	0.90	1.99	1.19
2	0.61	0.83	1.72	1.06
3	0.45	0.62	1.27	0.78
4	0.41	0.46	1.26	0.71
High	0.26	0.70	0.69	0.55
		High-Low		
Excess Return	-0.41***	-0.19	-1.30***	-0.63***
	(0.13)	(0.16)	(0.26)	(0.11)
CAPM Alpha	-0.39***	-0.23	-1.11***	-0.58***
	(0.13)	(0.16)	(0.25)	(0.11)
FF6 Alpha	-0.31**	-0.19	-1.42***	-0.64***
	(0.14)	(0.16)	(0.24)	(0.11)
HXZ Alpha	-0.27*	-0.14	-1.42***	-0.61***
	(0.14)	(0.17)	(0.25)	(0.11)

Table OA.VI

Two-way Portfolio Sorting on Patent Thickets and Product Market Competition

We conduct independent double sorts on patent thicket (*PT*) and one of the measures of product market competition into 15 groups based on the quintiles of PT at the end of year *t* – 1 and the 30th and 70th percentiles of product market competition at the end of year *t* – 1. Following Hou and Robinson (2006), we define sales competition (*SC*) as one minus the concentration of sales in an industry classified by the three-digit SIC code. Following Hoberg and Phillips (2016), product similarity (*PS*) measures how similar a focal firm's products are to those of its industry rivals, based on textual analyses of 10-K announcements. Following Hoberg et al. (2014), product fluidity (*PF*) captures how rivals change product words that overlap with the focal firm's vocabulary and therefore focuses on product space dynamics and changes in products. We construct the "Average" group, which averages the excess returns of the three control variable groups in the same PT group. We then construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each control variable group and hold these portfolios over the next twelve months (July of year *t* to June of year *t* + 1) to calculate monthly value-weighted returns. Sample period: July 1981 to June 2019 for SC, July 1997 to June 2019 for PS, and July 1998 to June 2019 for PF.

Panel A. Conditional Predictive Power on Sales Competition (SC)				
PT \ SC	**Low**	**2**	**High**	**Average**
Low	0.82	0.92	1.11	0.95
2	0.63	0.97	1.07	0.89
3	0.55	0.73	0.83	0.70
4	0.42	0.52	0.88	0.61
High	0.49	0.47	0.68	0.55
High-Low				
Excess Return	-0.33**	-0.45***	-0.44*	-0.41***
	(0.16)	(0.16)	(0.24)	(0.11)
CAPM Alpha	-0.35**	-0.35**	-0.41*	-0.37***
	(0.16)	(0.16)	(0.24)	(0.11)
FF6 Alpha	-0.17	-0.52***	-0.61***	-0.44***
	(0.16)	(0.16)	(0.22)	(0.10)
HXZ Alpha	-0.10	-0.55***	-0.66***	-0.44***
	(0.17)	(0.16)	(0.24)	(0.11)

Panel B. Conditional Predictive Power on Product Similarity (PS)				
PT \ PS	**Low**	**2**	**High**	**Average**
Low	0.44	0.87	1.44	0.92
2	0.27	1.11	1.79	1.06
3	0.55	0.39	1.32	0.75
4	0.27	0.54	0.82	0.54
High	0.45	0.58	0.53	0.52
High-Low				
Excess Return	0.01	-0.29	-0.91***	-0.39**
	(0.26)	(0.24)	(0.30)	(0.15)
CAPM Alpha	-0.24	-0.15	-0.81***	-0.40**
	(0.23)	(0.23)	(0.30)	(0.15)
FF6 Alpha	-0.03	-0.28	-0.82***	-0.38**
	(0.24)	(0.23)	(0.29)	(0.15)
HXZ Alpha	-0.06	-0.20	-0.87***	-0.38**
	(0.24)	(0.22)	(0.29)	(0.15)

Panel C. Conditional Predictive Power on Product Fluidity (PF)				
PT \ PF	**Low**	**2**	**High**	**Average**
Low	0.57	1.19	1.09	0.95
2	0.35	1.15	1.36	0.95
3	0.51	0.61	1.12	0.74
4	0.51	0.47	0.64	0.54
High	0.46	0.52	0.46	0.48
High-Low				
Excess Return	-0.11	-0.67**	-0.63**	-0.47***
	(0.22)	(0.30)	(0.30)	(0.17)
CAPM Alpha	-0.21	-0.48*	-0.53*	-0.41**
	(0.21)	(0.28)	(0.30)	(0.17)
FF6 Alpha	-0.07	-0.77***	-0.54*	-0.46***
	(0.22)	(0.26)	(0.29)	(0.15)
HXZ Alpha	-0.03	-0.67***	-0.56*	-0.42***
	(0.22)	(0.26)	(0.29)	(0.15)

Table OA.VII

Two-way Industry-Balanced Portfolio Sorting on Patent Thickets and Technological Complementarity among Patent Owners

We conduct independent double sorts on patent thicket (*PT*) and one of the measures of technological complementarity among previous patent owners into 15 groups based on the quintiles of PT at the end of year *t* – 1 and the 30th and 70th percentiles of technological complementarity among previous patent owners at the end of year *t* – 1. The ratio of mutual citations by previous patent owners (*MC1*) is defined as the ratio of all backward citations by previous patent owners that go to their counterparts in their historical patent portfolios. The ratio of mutually citing previous patent owners (*MC2*) is defined as the ratio of previous patent owners that cite their counterparts (i.e., previous patent owners that are cited by the focal firm) in their historical patent portfolios. We then construct a high-minus-low (High-Low) portfolio by holding a long (short) position in the top quintile (bottom quintile) PT portfolio within each control variable group and hold these portfolios over the next twelve months (July of year t to June of year *t* + 1) to calculate monthly value-weighted returns. Sample period: July 1981 to June 2019.

Panel A. Conditional Predictive Power on Mutual Citations by Patent Owners (MC1)				Panel B. Conditional Predictive Power on Mutually Citing Patent Owners (MC2)			
PT \ MC1	**Low**	**2**	**High**	**PT \ MC2**	**Low**	**2**	**High**
Low	0.76	0.88	0.88	**Low**	0.81	0.87	0.92
2	0.62	0.82	0.78	**2**	0.68	0.81	0.77
3	0.53	0.58	0.84	**3**	0.55	0.51	0.80
4	0.45	0.55	0.70	**4**	0.00	0.52	0.82
High	0.17	0.39	0.90	**High**	0.23	0.38	0.88
	High-Low				**High-Low**		
Excess Return	-0.59***	-0.50***	0.02	**Excess Return**	-0.58***	-0.49***	-0.04
	(0.22)	(0.18)	(0.16)		(0.20)	(0.18)	(0.16)
CAPM Alpha	-0.49**	-0.46**	0.01	**CAPM Alpha**	-0.51***	-0.44**	-0.04
	(0.22)	(0.18)	(0.16)		(0.20)	(0.18)	(0.16)
FF6 Alpha	-0.72***	-0.58***	0.31*	**FF6 Alpha**	-0.56***	-0.59***	0.25
	(0.22)	(0.18)	(0.16)		(0.20)	(0.18)	(0.16)
HXZ Alpha	-0.68***	-0.53***	0.27	**HXZ Alpha**	-0.57***	-0.54***	0.21
	(0.22)	(0.19)	(0.17)		(0.20)	(0.19)	(0.17)

Table OA.VIII
Fama-MacBeth Regressions

We run Fama-MacBeth regressions to examine the predictive power of patent thickets on future stock returns. In each cross section from 1981M7 to 2019M6, we regress the monthly stock returns in excess of one-month Treasury bill rates (in percentage) on within-industry ranked patent thicket (*PT*) in Panel A or simple ranked PT in Panel B by the fiscal year end. We also control for *Size* (in natural logarithm), *B/M* (in natural logarithm), *MOM11*, *REV*, *CTBE*, *CPPS*, *PQ*, *RDBE*, *MC1*, and *SC*, as described in Table II, as well as industry fixed effects. Industry fixed effect (Industry FE) is constructed using Fama-French 48 industry classifications. Standard errors (in parentheses) are computed following Newey and West (1987). ***, **, and * indicate significance levels of 1%, 5%, and 10%, respectively.

	Panel A: Within-Industry Ranked PT	Panel B: Simple Ranked PT
Patent Thicket (PT)	-0.03**	-0.04***
	(0.01)	(0.02)
Market Capitalization (Size)	-0.03	-0.03
	(0.05)	(0.05)
Book-to-Market Ratio (B/M)	0.41***	0.42***
	(0.07)	(0.07)
Momentum (MOM11)	-0.01	-0.01
	(0.20)	(0.20)
Reversal (REV)	-5.06***	-5.06***
	(0.46)	(0.46)
Patent Portfolio Size (CTBE)	-0.01	-0.01
	(0.05)	(0.05)
Counterparties' Patent Portfolio Size (CPPS)	-0.00***	-0.00***
	(0.00)	(0.00)
Patent Quality (PQ)	0.02**	0.02**
	(0.01)	(0.01)
R&D Intensity (RDBE)	0.36***	0.36***
	(0.13)	(0.13)
Mutual Citation (MC1)	-0.06	-0.07
	(1.45)	(1.45)
Product Compitition (SC)	0.17	0.17
	(0.19)	(0.19)
Observations	528,663	528,663
Number of Cross-Sections	456	456
R-squared	0.10	0.10
Industry FE	YES	YES

Table OA.IX

One-way Industry Balanced Portfolio Sorting on Patent Thickets Based on Forward Citations

We re-do the portfolio sorting analysis in Table VI with a measure of patent thickets based on forward citations. Specifically, this measure is defined as an adjusted value of one minus the concentration index of total forward citations that a previous patent owner receives. Sample period: July 1981 to June 2019.

PT	Excess Return Time-series Mean	CAPM Alpha	FF3 Alpha	FF5 Alpha	FF6 Alpha	HXZ Alpha
Low	0.91***	0.25**	0.30***	0.24**	0.26***	0.33***
	(0.23)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
2	0.79***	0.17**	0.18**	0.09	0.08	0.10
	(0.21)	(0.08)	(0.08)	(0.08)	(0.09)	(0.09)
3	0.65***	-0.00	0.03	0.01	0.03	0.07
	(0.22)	(0.08)	(0.08)	(0.08)	(0.08)	(0.09)
4	0.82***	0.17**	0.21***	0.20***	0.20***	0.24***
	(0.21)	(0.07)	(0.07)	(0.08)	(0.08)	(0.08)
High	0.56***	-0.09	-0.03	-0.01	0.00	0.07
	(0.21)	(0.07)	(0.06)	(0.06)	(0.06)	(0.07)
High-Low	-0.35***	-0.34***	-0.33***	-0.25**	-0.26**	-0.26**
	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)

Table OA.X

One-way Industry-Balanced Portfolio Sorting on Unexpected Patent Thickets

We re-do the portfolio sorting analysis in Table VI with a measure of patent thicket that excludes cited patents that are granted before the application dates of citing patents. This unexpected patent thickets measure is set to zero if computed as missing. Sample period: July 1981 to June 2019.

PT	Excess Return Time-series Mean	CAPM Alpha	FF3 Alpha	FF5 Alpha	FF6 Alpha	HXZ Alpha
Low	0.78***	0.13	0.15	0.09	0.13	0.13
	(0.22)	(0.10)	(0.09)	(0.10)	(0.10)	(0.10)
2	0.83***	0.20**	0.19**	0.05	0.06	0.13
	(0.21)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
3	0.65***	0.06	0.09	0.04	0.02	0.06
	(0.20)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)
4	0.76***	0.09	0.16**	0.20***	0.17**	0.22***
	(0.22)	(0.08)	(0.07)	(0.07)	(0.07)	(0.08)
High	0.56**	-0.10	-0.03	-0.00	0.03	0.08
	(0.22)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
High-Low	-0.22*	-0.23*	-0.19	-0.09	-0.10	-0.05
	(0.13)	(0.13)	(0.12)	(0.12)	(0.12)	(0.12)

Table OA.XI

One-way Industry-Balanced Portfolio Sorting on Patent Thickets Adjusted for Patent Transfers

We re-do the portfolio sorting analysis in Table VI with a measure of patent thickets that excludes transferred cited and citing patents. To construct such a PT measure of a focal firm at the end of year *t*, we exclude both i) the citing patents granted to the focal firm in the previous five years from year *t* – 4 to *t* but transferred to other firms/institutes by year *t* and ii) the cited patents (i.e., patents cited by the focal firm's citing patents that are granted in the previous five years) transferred to other firms/institutes by year *t*. This patent thickets measure adjusted for patent transfers is set to zero if computed as missing. The information about patent transfer is extracted from the USPTO patent re-assignment data 1970-2017. Sample period: July 1981 to June 2018.

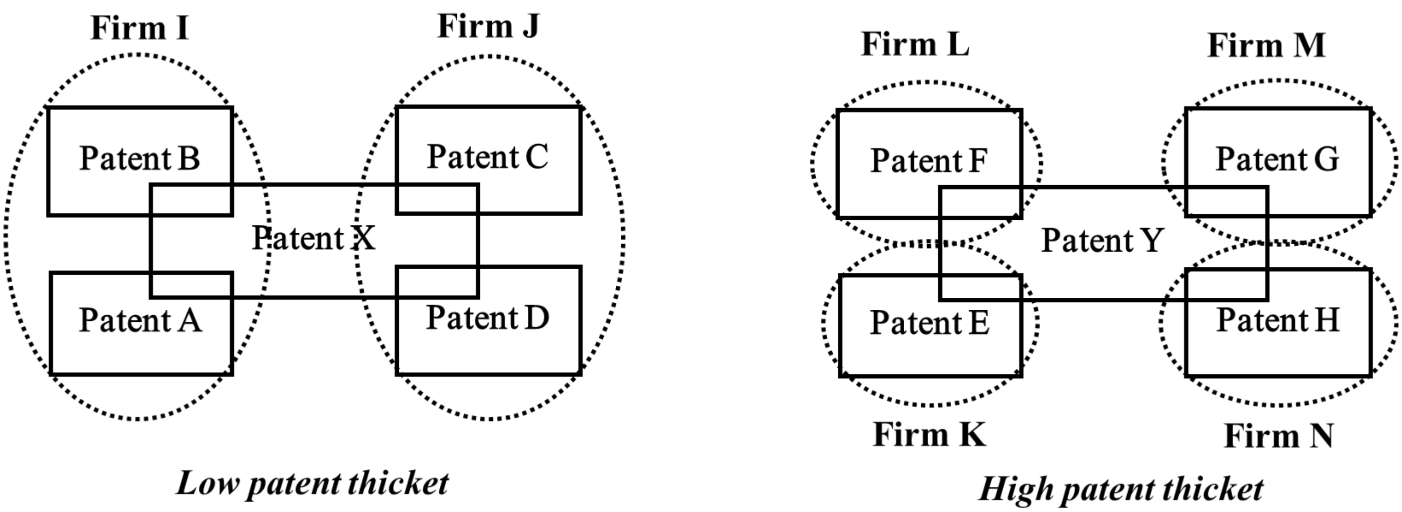
PT	Excess Return Time-series Mean	CAPM Alpha	FF3 Alpha	FF5 Alpha	FF6 Alpha	HXZ Alpha
Low	0.96***	0.30***	0.36***	0.34***	0.33***	0.40***
	(0.23)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
2	0.85***	0.20**	0.22***	0.16*	0.20**	0.23***
	(0.22)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)
3	0.62***	-0.01	0.04	0.01	-0.00	0.03
	(0.21)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
4	0.74***	0.09	0.15**	0.15**	0.11	0.16*
	(0.21)	(0.07)	(0.07)	(0.07)	(0.07)	(0.08)
High	0.55***	-0.09	-0.04	-0.02	0.01	0.07
	(0.21)	(0.07)	(0.06)	(0.07)	(0.07)	(0.07)
High-Low	-0.40***	-0.40***	-0.40***	-0.37***	-0.32***	-0.33***
	(0.12)	(0.12)	(0.11)	(0.12)	(0.12)	(0.12)

D Online Appendix Figures

Figure OA.I

Definition of Patent Thickets

This figure illustrates the concept of patent thickets (i.e., the fragmented ownership of overlapping patent rights). Each box denotes the claim coverage of a patent. Two patents (Patents X and Y) overlap with four other patents. The firm owning Patent X overlaps with Firms I and J in patent rights and is of lower patent thicket, and the firm owning Patent Y overlaps with Firms K, L, M, and N in patent rights and is of higher patent thicket.



E Proof of Model Propositions

Proof of Proposition 1. The market clearing condition gives Equation (8). Substituting the pricing kernel in Equation (7) into Equation (3), we have the optimal investment policy $p(x)$ defined by:

$$p(x_t) = E_t \left[\int_0^\infty e^{-\lambda s} \left(\frac{C_t}{C_{t+s}} \right)^\gamma (e^{-\delta s} e^{x_{t+s}}) ds \right].$$

Next, we replace C_{t+s} using the market clearing condition, i.e.,

$$\begin{aligned}
p(x_t) &= (C_t)^\gamma E_t \left[\int_0^\infty e^{-(\lambda+\delta)s} \frac{e^{x_{t+s}}}{(C_{t+s}/K_{t+s})^\gamma K_{t+s}^\gamma} ds \right] \\
&= C_t^\gamma E_t \left[\int_0^\infty e^{-(\lambda+\delta)s} \frac{e^{x_{t+s}}}{\left[e^{x_{t+s}} - \frac{1}{2} z \bar{L}_{t+s}\right]^\gamma K_t^\gamma exp\left[\int_0^s -\gamma\delta + \gamma z L_{t+\tau} d\tau\right]} ds \right] \\
&= \left[e^{x_t} - \frac{1}{2} z \bar{L}_t \right]^\gamma \phi(x_t),
\end{aligned}$$

in which the Feynman-Kac theorem implies that $\phi(x_t)$ satisfies the following equation:

$$[\lambda + (1-\gamma)\delta + \gamma z L_t]\, \phi(x_t) - A[\phi(x_t)] - e^{x_t} \left[e^{x_t} - \frac{1}{2} z \bar{L}_t \right]^{-\gamma} = 0.$$

Proof of Proposition 2. Using Equations (7) and (8), we first rewrite V_t as:

$$\begin{aligned}
V_t &= E_t \left[\int_0^\infty M_{t,t+s} D_{t+s} ds \right] \\
&= E_t \left[\int_0^\infty e^{-\lambda s} \left(\frac{C_t}{C_{t,t+s}} \right)^\gamma C_{t+s} ds \right] \\
&= C_t^\gamma E_t \left[\int_0^\infty e^{-\lambda s} \left(\frac{C_{t+s}}{K_{t+s}} \right)^{1-\gamma} K_{t+s}^{1-\gamma} ds \right] \\
&= \left(\frac{C_t}{K_t} \right)^\gamma K_t E_t \left[\int_0^\infty \left(\frac{C_{t+s}}{K_{t+s}} \right)^{1-\gamma} exp \left[\int_0^s -\lambda - (1-\gamma)\delta + (1-\gamma) z L_{t+\tau} d\tau \right] ds \right] \\
&= \left\{ e^{x_t} - \frac{1}{2} z \bar{L}_t \right\}^\gamma \psi(x_t) K_t,
\end{aligned}$$

in which following Feynman-Kac Theorem, $\psi(x_t)$ satisfies:

$$-\left[\lambda + (1-\gamma)\delta - (1-\gamma) z L_t\right] \psi(x_t) + \left[e^{x_t} - \frac{1}{2} z \bar{L}_t \right]^{1-\gamma} + A\left[\psi(x_t)\right] = 0.$$

Proof of Proposition 3. The proof directly follows the proof of Proposition 4 in Gomes et al. (2013).

Proof of Proposition 5. To compare the two, we have π_t^H and π_t^L:

$$\begin{aligned}\pi_t^L - \pi_t^H &= \frac{\sum_{f\in F_L} V_{ft}^o}{K_t^L p(x_t) + \sum_{f\in F_L} V_{ft}^o} - \frac{\sum_{f\in F_H} V_{ft}^o}{K_t^H p(x_t) + \sum_{f\in F_H} V_{ft}^o} \\ &= \left[\frac{\sum_{f\in F_L} V_{ft}^o}{K_t^L} - \frac{\sum_{f\in F_H} V_{ft}^o}{K_t^H}\right] \frac{K_t^H K_t^L p(x_t)}{(K_t^L p(x_t) + \sum_{f\in F_L} V_{ft}^o)(K_t^H p(x_t) + \sum_{f\in F_H} V_{ft}^o)},\end{aligned}$$

in which K_t^L (K_t^H) is the sum of all projects for firms with low (high) patent thickets. The sign of $\pi_t^L - \pi_t^H$ depends on the following:

$$\frac{\sum_{f\in F_L} V_{ft}^o}{K_t^L} - \frac{\sum_{f\in F_H} V_{ft}^o}{K_t^H}. \tag{14}$$

Consider $p(x_t) < p_H$. From the definition of V_{ft}^o, we have $\sum_{f\in F_H} V_{ft}^o = 0$, and hence, the sign of $\pi_t^L - \pi_t^H$ is always posisitve, indicating that $\beta_t^L > \beta_t^H$.

Consider $p(x_t) \geq p_H$. We then can substitute Equations (10) and (11) into Equation (14); that is:

$$\frac{\sum_{f\in F_L} V_{ft}^o}{K_t^L} - \frac{\sum_{f\in F_H} V_{ft}^o}{K_t^H} = \frac{V_t^o}{L_t N}\left[\frac{p(x_t)}{K_t^L/N_L} - \frac{p(x_t) - P_H}{K_t^H/N_H}\right].$$

As a result, we know $\beta_t^L \geq \beta_t^H$ if:

$$p(x_t) \leq \frac{K_t^L/N_L}{K_t^L/N_L - K_t^H/N_H} P_H.$$

Proof of Proposition 6. As $\lim_{\delta\downarrow 0}(L_t|_{P_H-\delta} - L_t|_{P_H}) = -\delta(1 - w_H) < 0$, we can see $\lim_{\delta\downarrow 0} V_{ft-}^{oL} > V_{ft+}^{oL}$. Using Equation (12), we prove $\lim_{\delta\downarrow 0} \beta_{t-}^L > \beta_{t+}^L$ by comparing π_t^i, which increases with V_{ft}^{oL}. By using our first part of the result, we obtain $\lim_{\delta\downarrow 0} \beta_{t-}^L > \beta_{t+}^L$.

F Numerical Simulation

To illustrate the implications of Propositions 5 and 6, we conduct an extensive numerical simulation to visualize the conditional CAPM we propose. We use the following parameters similar to Gomez et al. (2003): the time preference parameter $\lambda = 0.10$, the rate of project expiration $\delta = 0.03$, the long-term mean of the aggregate productivity $\bar{x} = 0.01$, the quality of investment opportunities $z = 0.50$, the volatility of the productivity variable $\sigma_x = 0.08$, the rate of mean reversion of the idiosyncratic productivity component $\theta_x = 0.275$, the rate of mean reversion of the idiosyncratic productivity component $\theta_\epsilon = 0.50$, the volatility of the idiosyncratic productivity component $\sigma_\epsilon = 2.00$, and the percentage of firms with high patent thickets $w_H = 10\%$. We use two values of the risk aversion coefficients $\gamma = 8$ and $\gamma = 11$, and the two values of the lower bound for the commercialization cost for firms with high patent thickets, $P_H = 0.08$ and $P_H = 0.10$.

Our simulation starts by solving the ODE functions in Propositions 1 and 2, and then derives the betas for aggregate assets in place and aggregate growth options (i.e., β_t^a and β_t^o). Next, we use the numerical simulation settings in Gomez et al. (2003) to generate an economy with 2,016 firms, in which 202 firms (about 10%) as firms with high patent thickets, and then continue the simulation until the economy reaches a steady state. With this steady state, we then repeat 100 times of simulation iterations for the economy at different aggregate productivity level x_t to find the portfolio betas for the firms with high and low patent thickets (i.e., β_t^H and β_t^L) based on Proposition 4.

Figure OA.II presents the following patterns: first, the beta of aggregate assets in place is much lower than that of aggregate growth options; in addition, the beta of aggregate assets in place (growth options) increases (decreases) with aggregate productivity (x_t). Second, the beta of the high-PT portfolio is much lower than that of the low-PT portfolio, which is consistent with Proposition 5. Third, the high-PT portfolio's beta increases with aggregate productivity, while the low-PT portfolio's beta slightly decreases with aggregate productivity. These findings are consistent with Proposition 6.

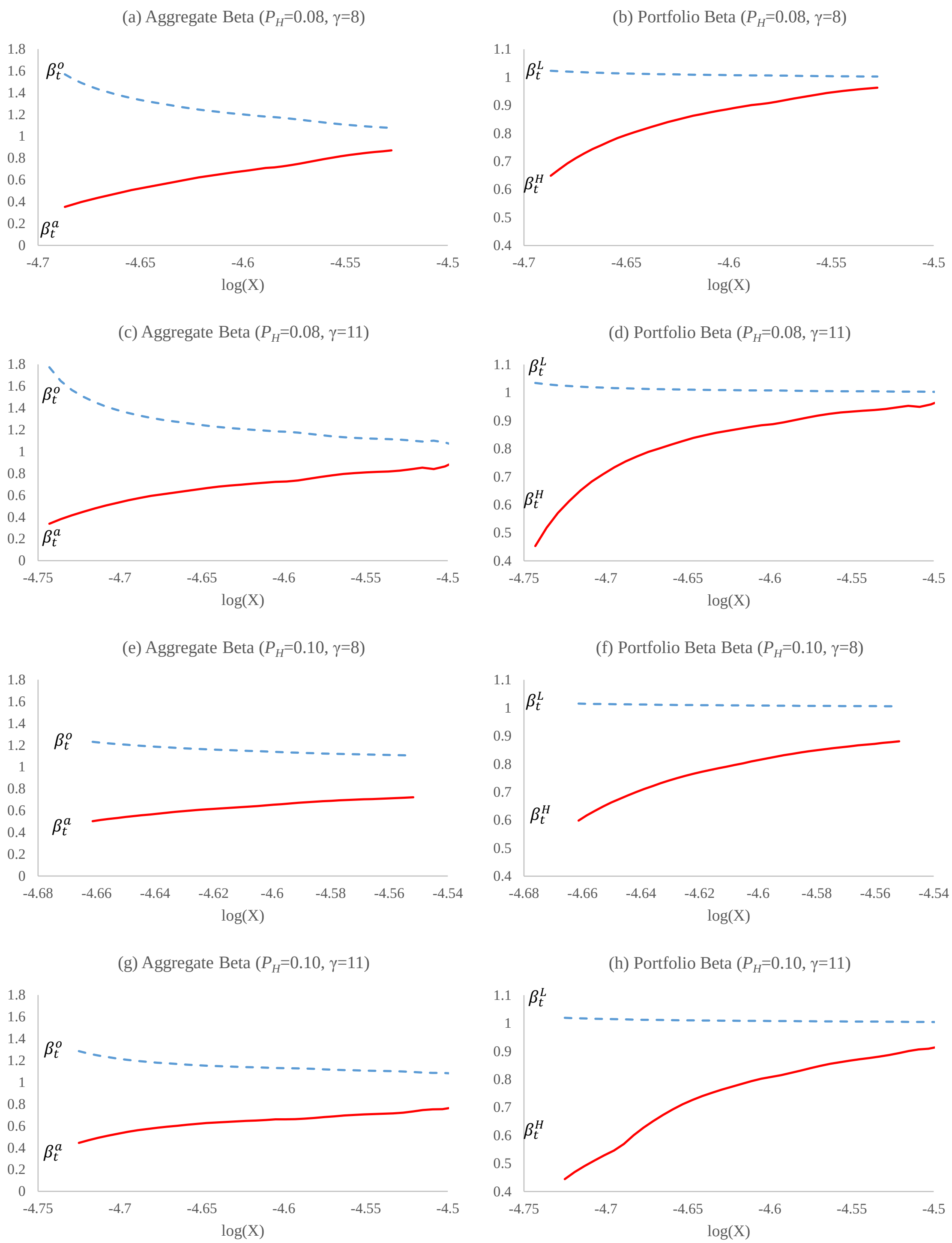


Figure OA.II: Aggregate and Portfolio Betas

In this simulation trial, we show the aggregate betas in the left panels, in which the solid line represents β_t^a and the dashed line represents β_t^o, and the portfolio betas in the right panels, in which the solid line represents β_H^a for firms with high patent thickets and the dashed line represents β_L^o for firms with low patent thickets.

G Additional Implications on Competition

Proposition OA.1 *We consider two almost identical economies A and B,*[1] *except that economy A is more competitive than economy B because there are more identical low-PT firms (i.e., $N_{LA} > N_{LB}$). The low-PT portfolio's market beta is lower in the more competitive economy ($\beta_t^{LA} < \beta_t^{LB}$) as long as the following condition is satisfied:*

$$p(x_t) \leq \frac{K_{ft}^L}{K_{ft}^L - K_{ft}^H} P_H, \tag{OA.17}$$

We note that the condition of Inequality (OA.17) is essentially identical to the condition of Inequality (13) in Proposition 5 when the number of low-PT and high-PT firms are constant (because $K_{ft}^L = K_t^L/N_L$ and $K_{ft}^H = K_t^H/N_H$). Thus, the condition is likely satisfied in a wide range of parameters. The intuition of this proposition is that competition reduces the weight of market risk of aggregate growth options in low-PT firms' betas, which leads to lower betas because $\beta_t^o > \beta_t^a$.

Proof. We substitute the definition of V_{ft}^{oL} in Equation (11) to π_t^L defined in Proposition 4, and then we have:

$$\pi_t^{Lj} = \frac{\frac{p(x_t)}{L_t}(1 - w_{Hj})V_t^o}{K_t^{Lj} p(x_t) + \frac{p(x_t)}{L_t}(1 - w_{Hj})V_t^o},$$

in which $j = \{A, B\}$ and $K_t^{Lj} = K_{ft}^{Lj} N_{Lj}$. When $p(x_t) < p_H$, we then have:

$$\pi_t^{Lj} = \frac{V_t^o}{K_t^{Lj} p(x_t) + V_t^o},$$

According to our assumption, we then have $K_t^{LA} > K_t^{LB}$, and, hence, $\pi_t^{LA} < \pi_t^{LB}$, as the same K_t^j and same x_t ensure that the same V_t^o . Following Equation (12), we then have $\beta_t^{LA} < \beta_t^{LB}$.

[1] To focus on the pure effect of competition, we require all low-PT firms in two economies to be of the same assets in place (i.e., commercialized patents) and the aggregate assets in place of two economies are the same (i.e., $K_{ft}^{LA} = K_{ft}^{LB} \equiv K_{ft}^L$, $K_{ft}^{HA} = K_{ft}^{HB} \equiv K_{ft}^H$, and $K_t^A = K_t^B \equiv K_t$). Also, at time t, the two economies are under the same x_t.

When $p(x_t) \geq p_H$, we have:

$$\pi_t^{Lj} = \frac{1}{1 + \frac{K_t^{Lj}(p(x_t) - w_{Hj}P_H)}{(1-w_{Hj})V_t^o}}, \tag{OA.18}$$

and the comparison between π_t^{LA} and π_t^{LB} depends on the term in the denominator, which can be rewritten as:

$$\frac{K_t^{Lj}(p(x_t) - w_{Hj}P_H)}{(1 - w_{Hj})V_t^o} = \frac{K_{ft}^L p(x_t)}{V_t^o} \frac{N_{Lj}(1 - w_{Hj}P_H/p(x_t))}{1 - w_{Hj}}.$$

The first part, $(K_{ft}^L p(x_t))/V_t^o$, remain the same for the two economies, and hence, the comparison then depends on:

$$\frac{N_{Lj}(1 - w_{Hj}P_H/p(x_t))}{1 - w_{Hj}} = (N_{Lj} + N_{Hj}) \left(1 - \frac{N_{Hj}}{N_{Lj} + N_{Hj}} \frac{P_H}{p(x_t)}\right). \tag{OA.19}$$

As K_t remains the same for both economies, we have:

$$N_{Hj} = \frac{K_t - N_{Lj}K_{ft}^L}{K_{ft}^H}.$$

We then substitute N_{Hj} into Equation (OA.19), calculate the first-order derivative with respect to N_{Lj}, and we find that when:

$$p(x_t) < \frac{K_{ft}^L}{K_{ft}^L - K_{ft}^H} P_H,$$

then we have Equation (OA.19) increases with N_{Lj}. Using Equation (OA.18), then we know π_t^{Lj} decreases with N_{Lj}, thereby yielding $\pi_t^{LA} < \pi_t^{LB}$, and hence, $\beta_t^{LA} < \beta_t^{LB}$. ■